A MEASUREMENT OF THE MUON MAGNETIC MOMENT USING COSMIC RAYS

1. Abstract

The muon magnetic moment is being measured via the decay of polarized cosmic-ray muons in a 44 G magnetic field. One, thick, 102.0 x 20.6 x 5.4 cm plastic scintillator detector was placed between two, 101.5 x 20.6 x 1.6 cm detectors in the uniform magnetic field produced by a solenoid. A veto-scintillator eliminated events from regions of non-uniform magnetic field. The time difference between when the muon stopped in the center detector and the detection of the decay positron was recorded for several thousand events. The decay positron is emitted along the direction of the precessing muon spin axis.

D. Ely, D. Kroening, and M. Yuly.
Department of Physics
Houghton College
One Willard Avenue
Houghton, NY 14744

2. Introduction

The precession of muons in a magnetic field has been of great interest in the past as a test of quantum electrodynamics (QED). Muon decay can also be used to explore relativistic effects such as time dilation and length contraction.

Cosmic ray muons (μ^+) are positively charged leptons with a mean lifetime of about 2.20 microseconds. They are created in the upper atmosphere in pion (π^+) and kaon (K^+) decays, and subsequently decay into positrons via

$$\mu^+ \rightarrow e^+ + \nu + \bar{\nu}$$

The muons have a net polarization as shown in **Figure 1**.

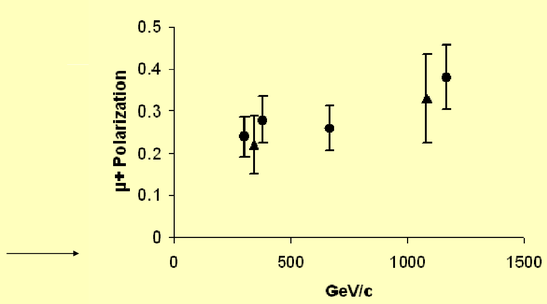

Figure 1. Plot of μ+ Polarization as a function of their energy. Obtained from "The Determination of the Muon Magnetic Moment from Cosmic Rays" (1974) by C. Amsler of the Laboratory of High Energy Physics, Swiss Federal Institute of Technology, Zurich, Switzerland

This experiment measures the magnetic moment of the muon by recording the lifetimes of muons as their spins precess in a uniform magnetic field. As seen in **Figures 2 and 4**, an incident muon enters the field, passes through scintillator 1, and stops in scintillator 2. The field causes the muon's spin to precess at a rate proportional to the field strength. Upon decay, a positron is emitted antiparallel to the muon's spin direction. Only positrons emitted in the direction of scintillator 3 were recorded. This allowed measurement of the lifetimes of muons whose spin were pointing up at the time of decay. As seen in **Figure 3**, since the muon's spin precesses at a constant rate, more events will be recorded for certain lifetimes—events corresponding to muon decay into a positron that hits scintillator 3. This causes the decay curve for the muon to vary sinusoidally.

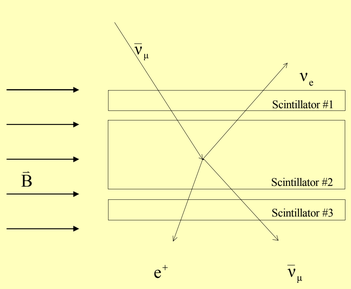

Figure 2. A positive muon deposited energy in scintillator 1 and stopped in scintillator 2. The muon's spin precessed about the axis of the magnetic field until it decayed into a positron and two neutrinos. A positron detected by the lower scintillator indicated that the muon's spin was pointing upwards at the time of decay.

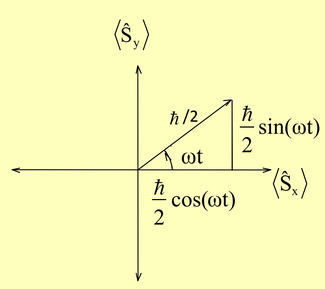

Figure 3. The expectation value of the muon will precess with angular frequency ω. The magnitude of the expectation value for the spin is /2. ħ

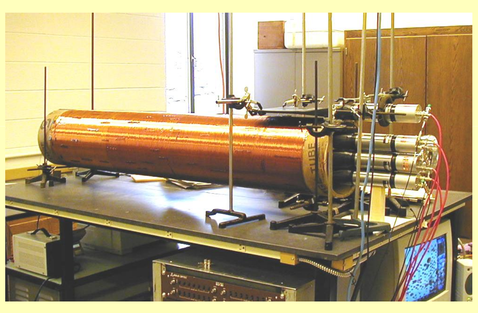

Figure 4. The large solenoid produced the constant magnetic field the muons precessed in. The three scintillators inside the solenoid detected the muons and decay positrons. The small scintillator over the end of the solenoid vetoed events in the non-uniform region of the field.

3. Theory

The Hamiltonian, or energy operator, may be written in terms of the magnetic moment operator.

$$\hat{H} = -\hat{\mu} \cdot \vec{B}$$

where $\hat{\mu} = \frac{gq\hat{S}}{2mc}$ and $\vec{B} = B_z\hat{k}$. Therefore, if $\omega = \frac{-gqB_z}{2mc}$

$$\hat{H} = \frac{-gq\hat{\vec{S}} \cdot \vec{B}}{2mc} = \frac{-gq\hat{S}_z B_z}{2mc} = \omega\hat{S}_z$$

The eigenvalues for $\hat{S}_z$ are $\pm\hbar/2$, so

$$\hat{H}|\pm z\rangle = \pm\frac{\omega\hbar}{2}|\pm z\rangle = \omega\hat{S}_z|\pm z\rangle$$

Consider a muon in a spin state $|\psi(0)\rangle$, perpendicular to the magnetic field

$$|\psi(0)\rangle \equiv |+x\rangle = \left(\frac{|+z\rangle}{\sqrt{2}} + \frac{|-z\rangle}{\sqrt{2}}\right)$$

At a later time, the state will evolve.

$$|\psi(t)\rangle = e^{-i\hat{H}t/\hbar}\left(\frac{|+z\rangle}{\sqrt{2}} + \frac{|-z\rangle}{\sqrt{2}}\right) = \left(\frac{e^{i\omega t/2}|+z\rangle}{\sqrt{2}} + \frac{e^{-i\omega t/2}|-z\rangle}{\sqrt{2}}\right)$$

One may also find the components of the spin in the x-y plane:

$$\langle +x|\psi(t)\rangle = \left(\frac{\langle +z|}{\sqrt{2}} + \frac{\langle -z|}{\sqrt{2}}\right)\left(\frac{e^{i\omega t/2}|+z\rangle}{\sqrt{2}} + \frac{e^{-i\omega t/2}|-z\rangle}{\sqrt{2}}\right)$$

and therefore $|\langle +x|\psi(t)\rangle|^2 = \cos^2\left(\frac{\omega t}{2}\right)$ and

$$|\langle -x|\psi(t)\rangle|^2 = \sin^2\left(\frac{\omega t}{2}\right).$$

It follows that $\langle \hat{S}_y \rangle = \frac{\hbar}{2}\sin(\omega t)$

and $\langle \hat{S}_x \rangle = \left(\frac{\hbar}{2}\right)\cos^2\left(\frac{\omega t}{2}\right) + \left(\frac{-\hbar}{2}\right)\sin^2\left(\frac{\omega t}{2}\right) = \frac{\hbar}{2}\cos(\omega t)$

This probability is time dependent and the expectation value will precess about the z-axis with angle ωt just as the magnetic moment would classically (**Figure 3**). Our setup is capable of detecting the incoming muon and the subsequent decay positron if it is incident on the lower scintillator. Because the decay positron will most often be emitted anti-parallel to the spin of the parent muon, the decay time measured by the lower scintillator will reflect the muon's final spin state.

The muon decay rate is given by:

$$R(t) = R_0 e^{\frac{-t}{\tau}} + B \qquad \text{(Equation1)}$$

Where τ, the mean lifetime, is 2.2 μs. The measured rate for the muon decay into a positron incident on the lower scintillator will therefore be:

$$R(t) = R_0 e^{\frac{-t}{\tau}}\left(1 + A\sin(\omega t + \delta)\right) + B \qquad \text{(Equation 2)}$$

4. Apparatus

Two plastic Bicron BC-400 scintillators of dimensions 101.5 x 20.3 x 1.6 cm, and one of 102.0 x 20.3 x 5.4 cm are placed in a 25.1cm inside diameter solenoid with B-field of 44 G. A 21cm long detector is placed above the coil to veto events in the non-uniform field region. The magnet consists of 1348 windings in a single layer of 19 gauge copper wire, and requires a potential of 120V to produce a 3.5A current (**Figure 4 and 5**). A graph of the magnetic field as a function of position in the solenoid is shown in **Figure 6**.

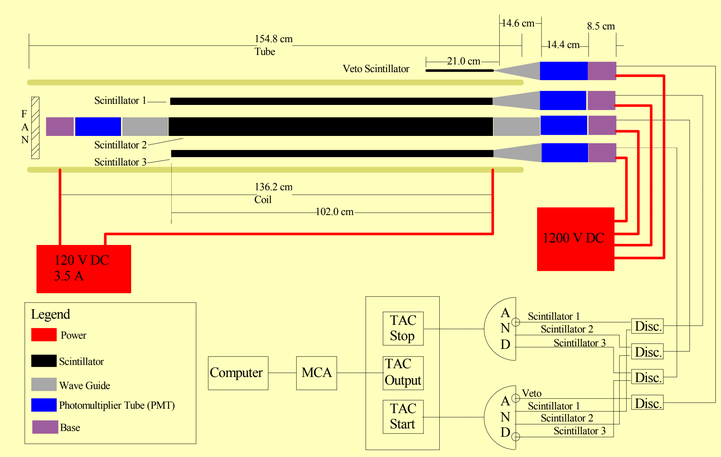

Figure 5. This is a schematic diagram of the tabletop apparatus and electronics.

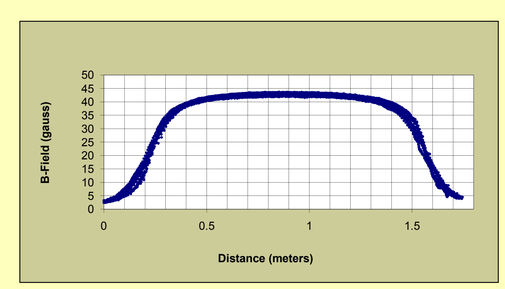

Figure 6. This is a plot of the magnetic field strength of the solenoid as a function of position inside the solenoid.

4. Results

Data have been collected for 40 days (~ 40,000 counts), and data acquisition is planned through December in order to minimize statistical uncertainty. The data collected so far seem to show a sinusoidal variance (Figure 8), as compared to the background decay curve taken without a magnetic field (Figure 7; 28,170 counts). A fit of equation 2 to current data yields a mean lifetime of 2.11μs, and ω is also comparable to previous experimental results. When data collection is complete, τ and g, with corresponding uncertainty, will be calculated.

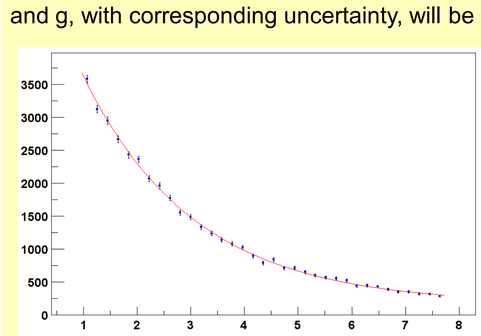

Figure 7. This is a plot of the background data, i.e., data collected with no magnetic field present. The x-axis corresponds to the muon lifetime in μs and the y-axis corresponds to the number of events. The solid line is the fit of Equation 1.

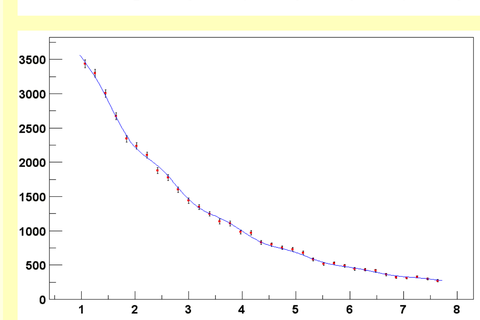

Figure 8. This is a plot of the data with a 42 G ± 5% magnetic field present. The axes are the same as in Fig. 4.1. The solid line is a fit of Equation 2.