

**A WIND DRIVEN POWER  
GENERATING SYSTEM: PRELIMINARY  
MODELING OF POWER CURVES**

By

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A thesis submitted in partial fulfillment of the  
requirements for the degree of

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## **Abstract**

Most power curves for current wind power systems depend on field measurements. Relationships between the details of the design and power output are therefore often hidden. The overall goal of this project is to design and build a low cost, mechanically efficient wind driven power-generating system in the 1 to 5 kilowatt range and to predict and improve its performance based on equations derived from fundamental principles. A series of theoretical power curves are presented that successively incorporate greater complexity of blade parameters. The aim is to optimize each individual parameter, then design an efficient blade design based on those specifications.

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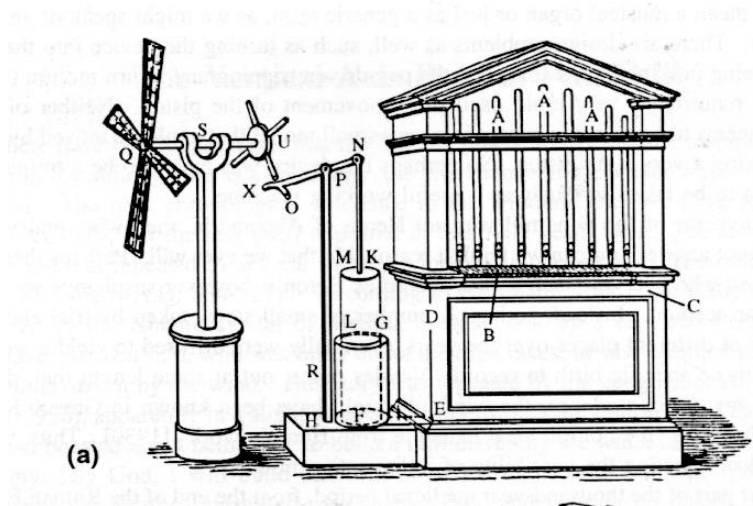
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## INTRODUCTION AND BACKGROUND

### 1.1 History of wind power

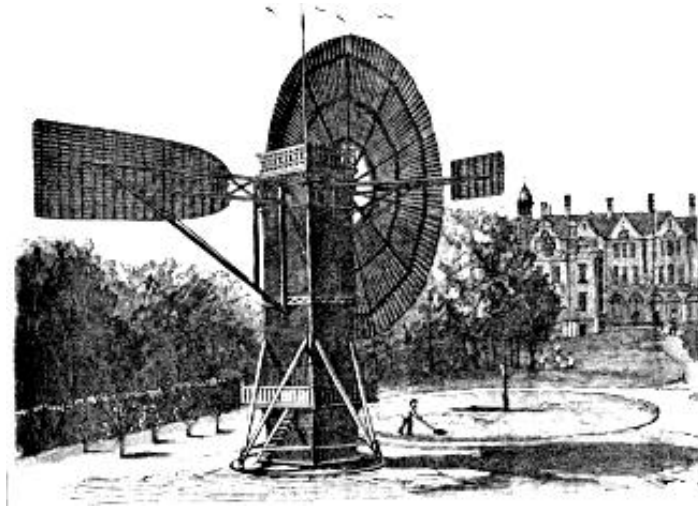
Wind energy may be the oldest form of applied power extraction used by man. The first extensive use of windmills is often attributed to Persia, or modern-day Iran. Farmers in the southern Iranian province of Sīstān still use vertical-axis drag windmills based on ancient designs to mill grain [1]. These machines generated power using drag forces on six to ten blades mounted on a vertical axis. Up to 50 of these primitive windmills are often built in a line to catch the prevailing wind just outside the town, and each can mill up to one ton of grain in 24 hours. By around 60 A.D. windmills were being used for recreational purposes. One of the earliest records and sketches of a windmill is from Hero of Alexandria's *Treatise on Pneumatics* [2], which describes in detail an organ that uses a windmill to pump air through the pipes as seen in Figure 1.



**Figure 1.** Heron of Alexandria's windmill. The windmill shaft moves a piston that pumps air through the organ pipes. Illustration from Ref. [2].

Wind power application was limited to mechanical tasks until electricity became the primary source of power. The first electricity-generating windmill, shown in Figure 2, was invented and built by Charles Brush in Cleveland, Ohio in the 1880's [3]. The rotor of Brush's

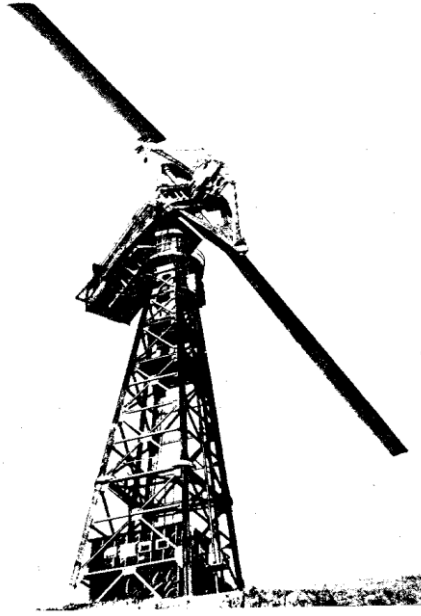
“Windmill Dynamo” spanned 17 m. in diameter and was comprised of 144 twisted blades. The dynamo, or generator, could produce up to 12 kW. Brush used the power to charge batteries that were a constant power source for the 350 incandescent lights in his house. The batteries provided between 70 and 75 volts.



**Figure 2.** Brush's "Windmill Dynamo". Built ca. 1890, the windmill powered the 350 lights in his home.

Illustration from Ref. [3].

The first turbine to synchronously feed power to a utility network was the Smith-Putnam turbine erected in 1941 at Grandpa's Knoll in Vermont [4]. The turbine had a rotor diameter of 53.3 m, a tower height of 36.5 m, and a power rating (maximum power output) of 1.25 MW, enough power to light a town at that time. The project was abandoned in 1945 after two blade failures lead to the termination of funding.



**Figure 3.** Putnam's wind turbine. This turbine was built in 1941 and had a rating of 1.25 MW. Picture from Ref. [4].

Power ratings for current industrial wind turbines typically range from 50 KW to 5000KW for turbines diameters of 15 m to 112 m, respectively, as seen in Table 1 [5]. In 2001, the total power capacity world wide in installed wind turbines was over 20,000 MW [5].

**Table 1.** Generalized height, diameter, and power rating of current wind turbines. Figures from Ref. [5].

Rated capacity (kW):	50	300	750	1000	2000	5000
Rotor diameter (m):	15	34	48	60	72	112
Tower height (m):	25	40	60	70	80	100

## 1.2 The benefits of using wind power

According to a 1976 report to the National Science Foundation [6], the wind is a clean, free, renewable natural resource. As demand for energy in today's society continues to increase, the need for renewable energy sources becomes more evident. Benefits of wind energy include its minimal effect on the environment and its overall low cost. Though the emission of greenhouse gasses may be associated with the fabrication and construction of turbine

components, the actual operation does not discharge any potential harmful gasses. Although the reduced wind velocity behind the turbine changes the microclimate surrounding the turbine, it has been shown that these minute changes have little or no effect on the immediate or regional environment. There has been concern about turbines increasing bird mortality due to the amount of birds that fly into the rotor. However, interference with local and migratory birds can easily be avoided by not positioning turbines in areas of high population density for the birds. For most migratory birds, wind turbines are also not a problem because the birds usually fly at altitudes of at least 150 m, and the highest wind turbines currently are around 125 m high.

In terms of cost, wind turbines have a relatively fast ‘pay back’ time. The time it takes for the turbine to return the costs of production and installation can be as quick as six months [7]. Also, for a medium turbine (100-500 kW), the ratio of energy input, or energy used in production and set up, to energy output, or energy produced by the turbine, can be as low as 1/5 at sites with appropriate wind conditions [8]. Installation is also relatively fast. A medium sized industrial turbine can be installed and operating typically one year from contract [7]. As seen in Table 2, costs for electricity produced from wind power is competitive with other renewable and non-renewable sources and, for the most part, less expensive overall [9].

**Table 2.** Comparative costs of electricity sources. Cost varies within range presented from various methods of generation using a particular source. Hydro, solar, and wind electricity costs also vary due to natural fluctuations in water flow, sun light, and wind, respectively. Figures from Ref. [9].

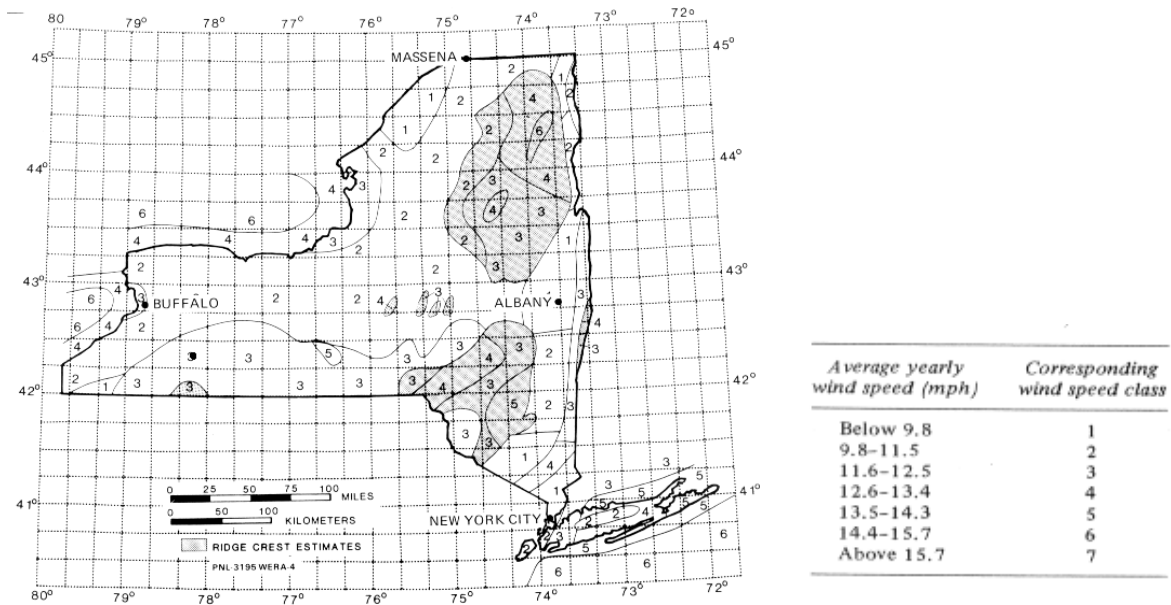
Energy Source	Cost (cents/kWh in 1993)
Coal	4.6 - 5.6
Gas	3.9 - 7.0
Hydro	2.9 - 11.3
Biomass	5.8 - 12.2
Nuclear	10.8 – 14.3
Solar	6.7 - 8.0
Wind	3.3 - 4.6



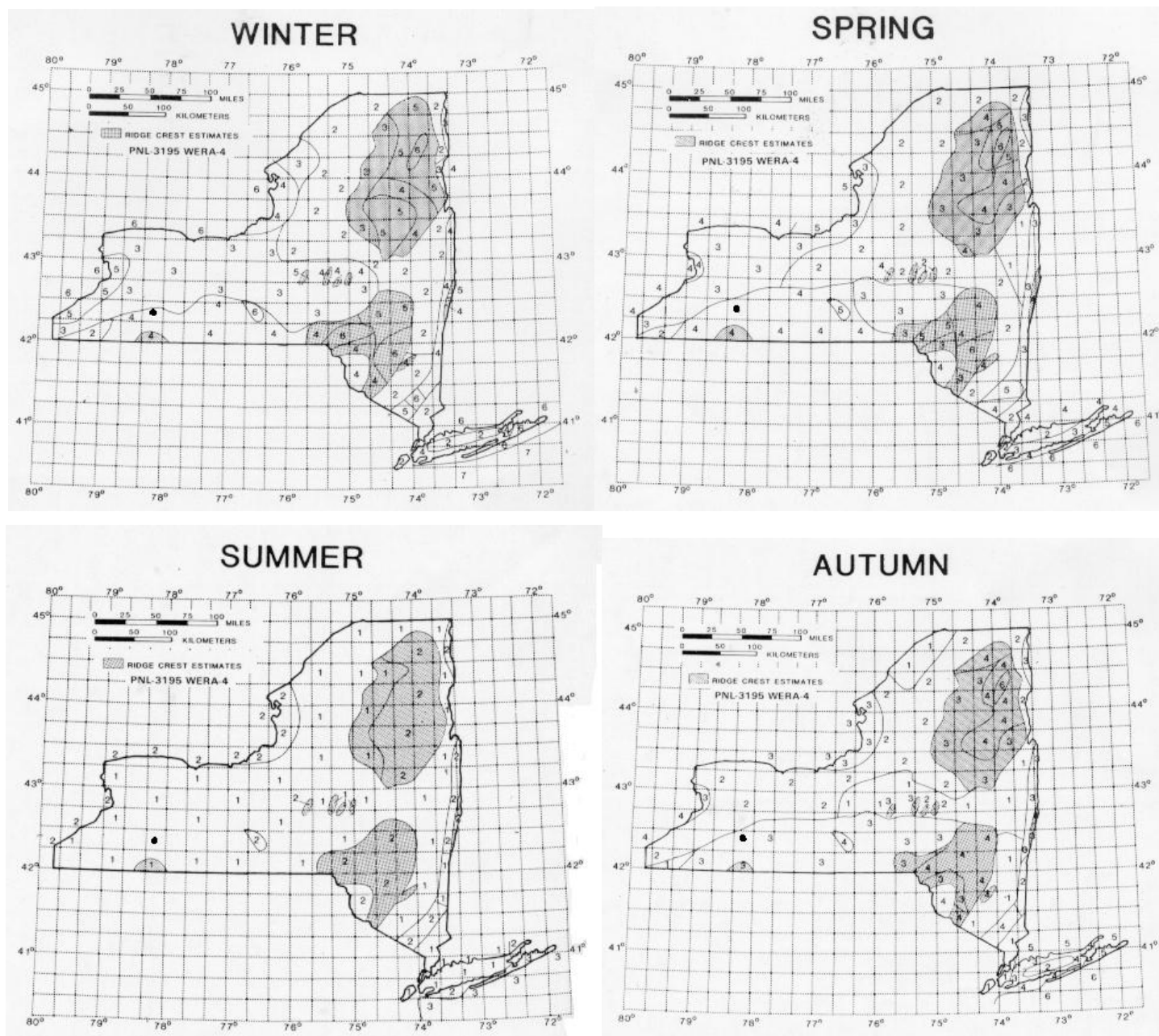
### 1.3 Wind resource and location

The United States wind resource is large enough to produce to 4.4 trillion kWh of electricity each year, or about 20% of the country's energy needs. About 90% of this resource is located in Montana, Minnesota, and North Dakota, and Texas [10]. It is notable that this location is also advantageous ecologically because most bird migration routes are along the west coast, Appalachian range, and eastern seaboard [11].

Wind ratings, or wind speed classes, are used to describe the energy contained in the wind in a given area at a particular altitude, and are based on mean wind speeds over a season or year. Wind ratings range from 1 (mean wind speed is below 9.8 m/s at an altitude of 10 m) to 7 (mean wind speed is above 15.7 m/s at 10 m). Areas with wind ratings of 3 or higher are considered to be appropriate for wind power development. Locations with ratings of 6 or 7 are typically offshore, and in most areas ratings vary seasonally [4]. Based on the wind rating scale, western New York is just on the border of being a good potential location for wind energy production. As seen in Figure 4 and Figure 5, wind ratings in the area surrounding Houghton, New York range from 1 in the summer to 4 in the spring and winter, with an average wind rating of 3 [12].



**Figure 4.** Annual wind ratings for the State of New York and the wind speeds associated with each speed class. Houghton, NY's location, about 42.42° N Lat. 78.16° W Long., is indicated with a black dot. Illustration from Ref. [12].

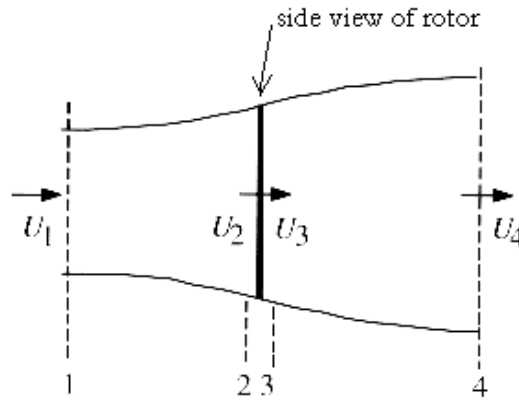


**Figure 5.** Seasonal wind ratings for the State of New York. Houghton, NY's location, about 42.42° N Lat. 78.16° W Long., is indicated with a black dot. The scale for the wind rating is in Figure 4. Illustrations from Ref. [12].

#### 1.4 Previous blade design and performance descriptions

Two primary methods have been previously used to determine the relationships between wind speed, blade design, and power output: 1) the use of a power coefficient to describe rotor efficiency, and, 2) the trial and error testing of blade designs. Both methods are, in some part, empirically based, and neither clearly or directly relate blade parameters to rotor power output via fundamental principles. Some important blade parameters include the angle of attack, or the angle the blade makes with the wind (as seen relative to the blade), the length of the blade, the width of the blade, and the angular velocity of the rotor.

The efficiency of a turbine rotor is typically described in terms of the power coefficient. The power coefficient is a fraction of wind's power that is extracted by the rotor, and is based on original calculations made by Albert Betz in 1926 [13]. Betz calculated the energy and power transferred from the wind to the rotor by comparing the kinetic energy of the wind before and after it passed through the area swept by the rotor. The wind has speed  $U_1$  before it meets the rotor, and a slower speed  $U_4$  after it passes through the rotor, as seen in Figure 6.



**Figure 6.** Betz's model of the wind's interaction with a turbine rotor. To maintain constant wind mass flow, the flow area of the wind increases when it slows down after passing through the rotor. Illustration from Ref.

[5]

According to Betz [5, 13], the power that is transferred to a rotor of swept area  $A_2$  is

$$P = \frac{1}{2} \rho A_2 (U_1^2 - U_4^2) U_2 \quad (1-1)$$

where  $\rho$  is the density of the air, and  $U_1$ ,  $U_2$ , and  $U_4$  are the speeds of the wind at positions 1, 2, and 4, respectively. This equation for power considers the total swept area and how much the wind is slowed down by the rotor, and not any insight into how particular blade parameters relate to the rotor efficiency. The power coefficient,  $C_p$ , is defined as

$$C_p = 4a(1-a)^2 \quad (1-2)[5]$$

where  $a$  is the fractional decrease in wind velocity due to the rotor called the “axial induction factor”, and is defined as

$$a = \frac{U_1 - U_2}{U_1} \quad (1-3)[5]$$

Combining Equations 1-1, 1-2, and 1-3 and solving for  $C_p$  gives

$$C_p = \frac{2P}{U^3 \rho A_2} \quad (1-4)[5]$$

where  $U$  is the speed of the wind before it is close enough to interact with the rotor.  $C_p$  is a function of the angle of attack and angular velocity, but rarely is an equation given that directly relates parameters and  $C_p$  [4, 5, 13, 14]. When such a function is given, it is usually said to describe a blade shape approximation, based on known performances of similar blades [5].

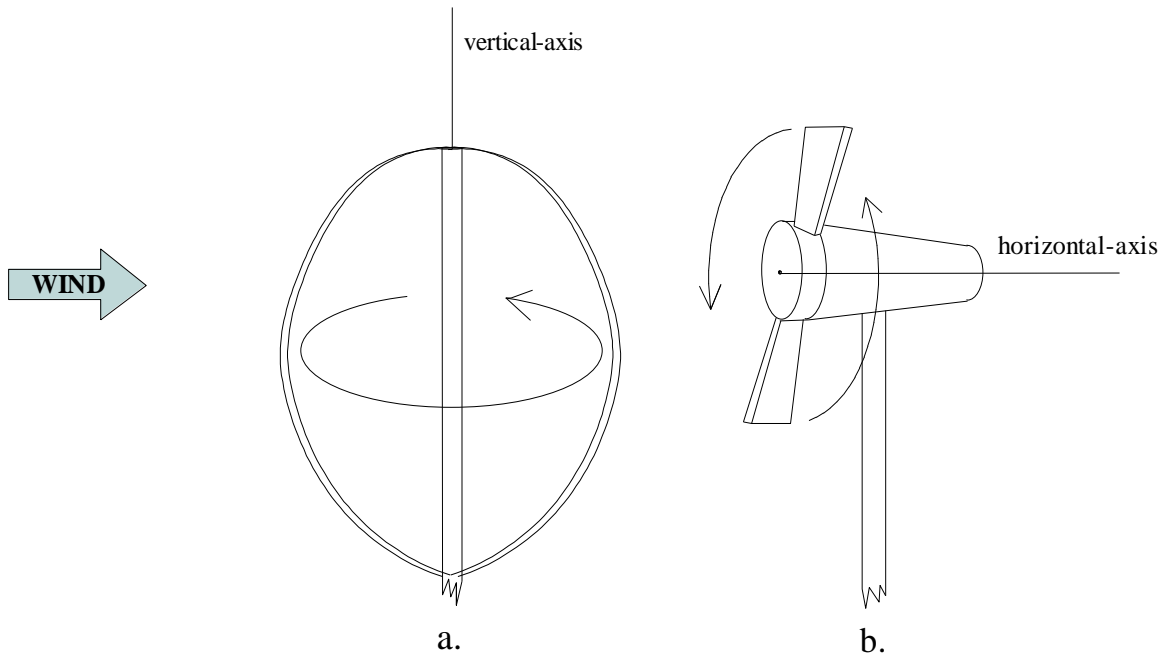
In the Smith-Putnam turbine project [4], basic calculations of power output based on rotor diameter, tower height, and average wind speed proved unsatisfactory in predicting the turbine’s power output. Four trials were made to determine an optimal blade design. Twenty-four blades were built and tested, each having different widths, twists, and tapering rates. Their performances were compared and the most efficient one was chosen. While the best design out of the 24 blades tested was found, the actual relationship between blade design, wind velocity, and power output was never explicitly found.

## 1.5 Project goals and parameters

The goal of this project is to design an efficient, inexpensive, 1-5 kW wind turbine to power a household and possibly sell power to energy companies. This thesis focuses on one aspect of this goal, namely, determining the relationship between the blade design, wind speed and power output from fundamental principles. Basic rotor parameters of axis orientation and the number of blades were determined before the details of blade design were investigated. .

### 1.5.1 Axis orientation

There are two general types of wind turbines: vertical-axis wind turbines (VAWTs) and horizontal-axis wind turbines (HAWTs). The rotor of a VAWT moves around a vertical axis so the effective surface (the surface that interacts with the wind) moves parallel to the wind's velocity. The only VAWT used commercially for electricity generation is the Darrieus model [15] seen in Figure 7a. A HAWT has a rotor shaft on a horizontal-axis and the effective surface is perpendicular to the wind velocity. The example in Figure 7b is mounted up-wind, meaning rotor is up-wind of the tower.

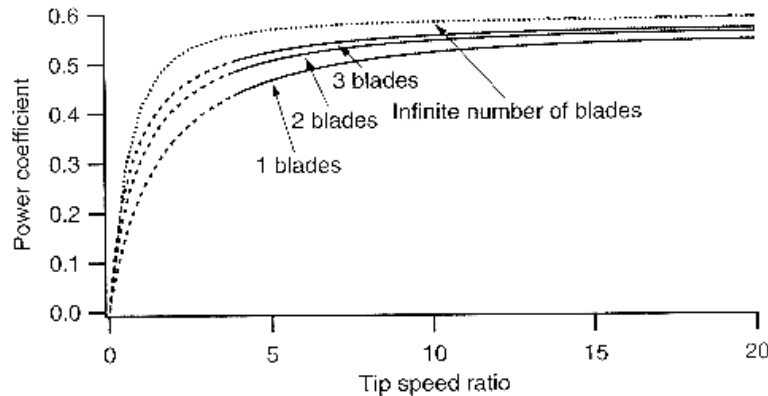


**Figure 7.** Turbine axis orientations. a) Darrieus style vertical-axis wind turbine. b) Two blade, horizontal-axis wind turbine.

There are advantages and disadvantages to either type. A VAWT can use wind from any side of the turbine, and thus does not require the yaw system used by HAWT's to stay at the right angle to the wind. Also, the generator on a VAWT may easily be placed close to the ground, whereas most HAWT generators are directly connected to the rotor shaft at the top of the tower to prevent mechanical power losses from connecting the shaft to a ground station. One disadvantage of VAWTs is that they use wind over the entire height of the turbine, while HAWT use just the wind near the top where wind velocities are much higher. Blades on a VAWT suffer fatigue damage faster than on a HAWT because they are typically much longer and have very small connections to the shaft. Also, more research has been done on HAWT performance [15]. Based on these comparisons and the relatively easier construction of a HAWT, the remainder of this thesis will focus on details of HAWT design.

### 1.5.2 Blade number

The number of blades on a turbine affects the turbine's power output and the stability of the whole system. Assuming no drag forces, as shown in Figure 8, power output increases slightly with the number of rotor blades. Thus the cost and efficiency of the blades determine how many are used.

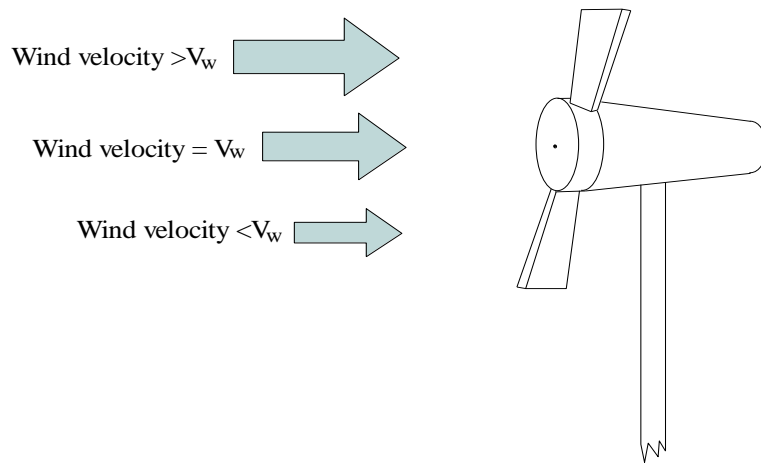


**Figure 8.** Comparison of maximum power coefficients for rotors with 1,2,3, and an infinite number of blades.

The tip speed ratio is the ratio of the blade tip speed to the wind speed. Illustration from ref. [5].

However, the drawback to both one and two bladed turbines is that they fatigue quickly from of wind shear. Wind shear is the term that describes how, because the wind has a higher velocity at higher elevations, it exerts more force on the blades while they are above

the rotor hub than when they are below it, as shown in Figure 9. One and two blade systems often include a teetering mechanism that allows the rotor to be pushed backward. This mechanism typically includes several moving shaft pieces, and can be difficult to install and repair [15]. Also, the teetering angle is limited to prevent the blades from hitting the tower and thus may sometimes not be enough for the wind conditions. A rotor with an odd number of blades is better than one with an even number because the distribution of the three blades around the hub distributes the force difference more evenly.



**Figure 9.** Windshear. The velocity of the wind is greater at higher elevations, and thus exerts more pressure on the blades when they are on top.

## Chapter 2

### DERIVATION OF POWER EQUATION

#### 2.1 Assumptions and definitions

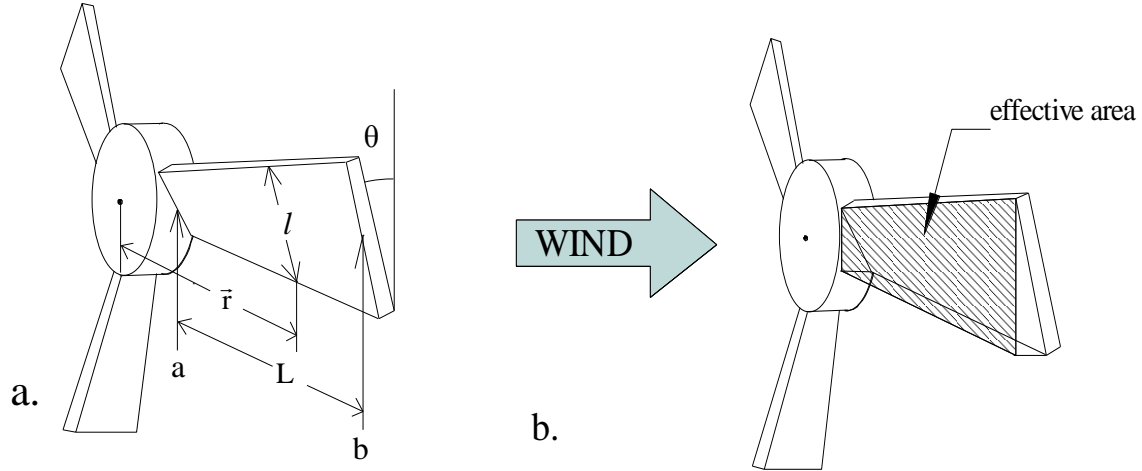
The derivation of the power equation is based on several assumptions and use particular terms to describe the blade parameters. Assumptions made in finding the power transferred from the wind to the turbine blade are that

1. the wind is considered to be blowing with a constant velocity,  $\vec{V}_b$ ,
2. friction between the blade and the generator shaft is not considered at this point,
3. the blade is assumed to experience no drag forces,
4. the wind is assumed to rebound from a perfectly elastic collision with the blade, and
5. the blades are considered to be infinitely thin and flat so there is no curvature along the width of the blade.

Useful blade parameters, shown in Figure 10, include the setting angle, the radius of the rotor, the length of the blade, the width of the blade, and the effective area of the blade.

The setting angle,  $\theta$ , is the angle the blade makes with the vertical axis. The total radius of the rotor is measured from the center of the hub out to the tip of the blade, and  $r$  is the radius of a particular point along the blade. The length of the blade,  $L$ , is measured from the point at which the blade connects to the rotor hub and extends to the tip of the blade, or from point a to point b in Figure 10a. The width of the blade is the measurement perpendicular to the blade length at any point along the length. The effective area of the blade is the part of the blade area that is perpendicular to the movement of the wind, as seen in Figure 10b.





**Figure 10.** Blade parameters. a)  $\theta$  is the setting angle,  $l$  is the width of the blade, the length,  $L$ , extends from point a to b, and the radius is measured from the center of the hub out along the length. b) The effective area is the area of the blade perpendicular to the velocity of the wind.

## 2.2 Initial power equation

To find the power transferred from the wind to the blade it is helpful to first consider an infinitely small segment of one blade. The overall blade has constant angular velocity and uniform setting angle, width, and effective area. As seen in Figure 11, a small segment of the blade at a distance  $\vec{r}$  from the center of the rotor and with a length of  $dr$ , has velocity  $\vec{V}_b$  in the positive  $y$  direction at time  $t$ . The wind has velocity  $\vec{V}_w$  in the positive  $x$  direction. The relative wind, or the wind as seen in the rest frame of the blade segment, has velocity  $\vec{V}_r$ . The small air mass,  $dm$ , hits the blade segment, and rebounds elastically at a velocity  $\vec{V}'$  relative to the blade, which has the same magnitude as  $\vec{V}_r$ .

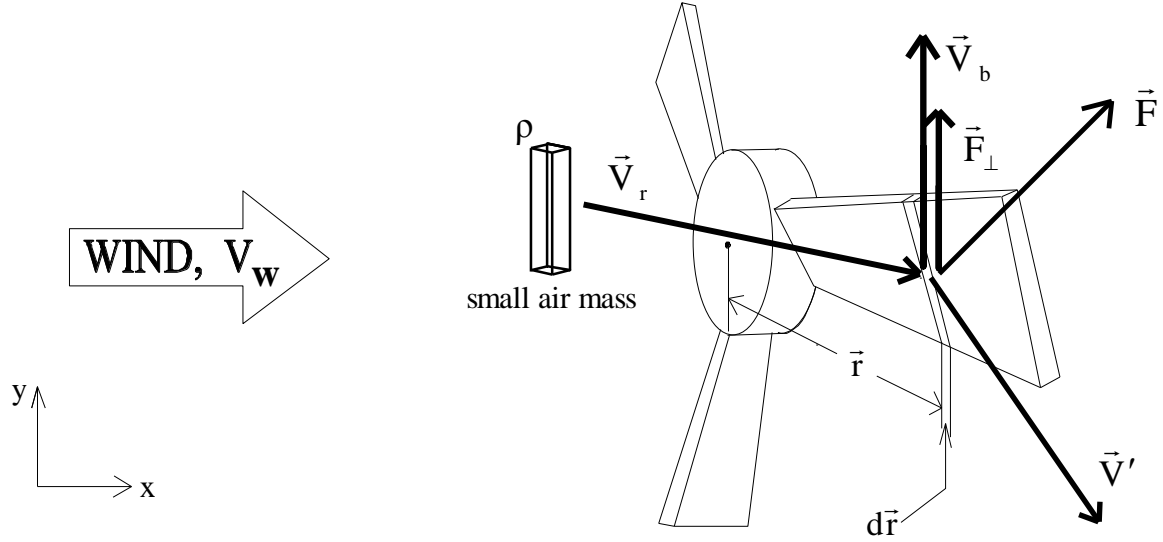
The power supplied to the rotor,  $P$ , and the torque,  $\vec{\tau}$ , on the rotor shaft can be written

$$P = \vec{\omega} \cdot \vec{\tau} \quad \text{and} \quad \vec{\tau} = \vec{r} \times \vec{F} \quad (2-1)(2-2)$$

where  $\vec{\omega}$  is the angular velocity of the blade,  $\vec{\tau}$  is the torque applied to the blade by the wind,  $\vec{F}$  is the force of the wind on the blade, and  $\vec{r}$  is the distance from the center of the rotor to the point on the blade where the force is acting. Only the component of the force

that is perpendicular to the radius is needed since the blade only moves in the vertical plane, so torque and power are simply

$$\tau = rF_{\perp} \quad \text{and} \quad P = \omega rF_{\perp}. \quad (2-3)(2-4)$$



**Figure 11.** Small air mass and blade segment. A small air mass,  $dm$ , of density  $\rho$  and initial velocity  $\vec{V}_r$  relative to the blade elastically collides with the small blade segment and rebounds at a relative velocity  $\vec{V}'$ . The small blade segment of width  $dr$  is a distance  $r$  from the center of the rotor and a constant velocity  $\vec{V}_b$  in the positive  $y$  direction.

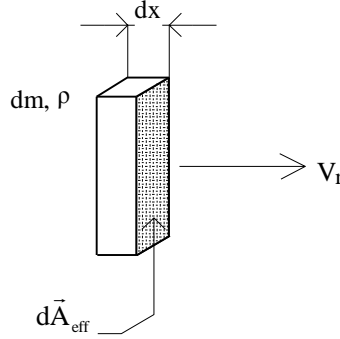
The force of the blade segment on a small air mass may be found by first considering Newton's Second Law,

$$d\vec{F} = \frac{d\vec{p}}{dt} \quad \text{or,} \quad d\vec{F} = dm \frac{d\vec{v}}{dt} \quad (2-5)(2-6)$$

where  $\vec{p}$ ,  $dm$ , and  $\vec{v}$  are the momentum, mass, and velocity of the small air mass, respectively. Equation 2-6 may be rewritten as

$$\vec{F} = \frac{dm}{dt} d\vec{v}. \quad (2-7)$$

The term  $\frac{dm}{dt}$  is called the mass flow rate, or the rate at which the small air mass travels through the effective area perpendicular to the air mass' velocity. As shown in Figure 12, the mass,  $dm$ , of this small block of air is  $\rho dV_m$  where  $dV_m$  is the volume.



**Figure 12.** Small air mass of width  $dx$ , velocity  $\vec{V}_r$ , mass  $dm$ , density  $\rho$ , and effective area  $dA_{eff}$ .

Defining  $V_m = dA_{eff}$  gives

$$\frac{dm}{dt} = \rho dA_{eff} \frac{dx}{dt} \quad \text{or} \quad \frac{dm}{dt} = \rho dA_{eff} V_r \quad (2-8) \quad (2-9)$$

The force of the small air mass on the small blade segment is then the product of the mass flow rate and the change in the velocity of the air mass as seen by the blade segment:

$$dF_y = \frac{m}{dt} dV = \rho dA_{eff} V_r \Delta V_y \quad (2-10)$$

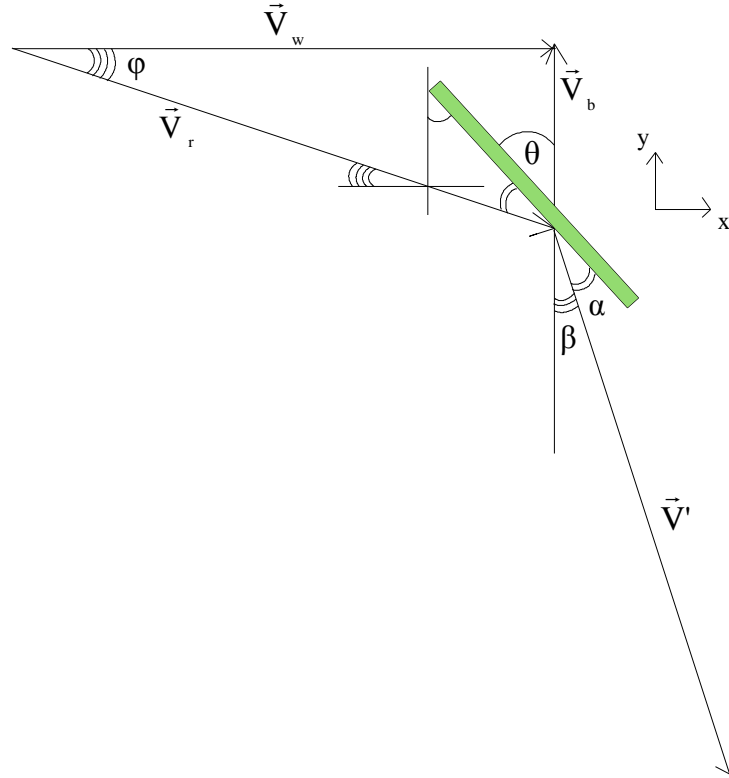
Note that only the change in the  $y$ -component of the velocity of the air mass is needed. The blades are only free to move in the vertical axis, so only the vertical, or  $y$ -component of momentum transfer is considered. The power transferred from the small air mass to the blade segment is then

$$dP = \omega r F_{\perp} = \omega r \rho dA_{eff} V_r \Delta V_y \quad (2-11)$$

The air mass' change in velocity,  $\Delta V_y$ , and the effective area,  $dA_{eff}$ , may be found using geometry and trigonometric relationships.

### 2.3 Finding the change in the wind velocity

$\Delta \vec{V}$  is the difference between the wind's velocity after it hits the blade and its velocity before it hits the blade. As shown in Figure 13, the angle the blade makes with the relative wind is  $\alpha$ , the angle of attack, and the angle the blade makes with the vertical axis is  $\theta$ , called the setting angle. As seen from the reference frame of the blade, the wind approaches with velocity  $\vec{V}_r$ , elastically collides with the blade, and then rebounds with velocity  $\vec{V}'$ .



**Figure 13.** The wind approaches with a relative velocity  $\vec{V}_r$ , and then rebounds with velocity  $\vec{V}'$ .  $\theta$  is the setting angle,  $\alpha$  is the angle of attack,  $\vec{V}_b$  is the velocity of the blade, and the wind has velocity  $\vec{V}_w$ .

Since the collision is elastic, the magnitudes of  $\vec{V}_r$  and  $\vec{V}'$  are thus equal and, as seen in Figure 13, can be written as

$$|\vec{V}_r| = |\vec{V}'| = \sqrt{V_b^2 + V_w^2} = V. \quad (2-12)$$

Broken down into components,  $\vec{V}_r$  and  $\vec{V}'$  are

$$\vec{V}_r = -V_b \hat{i} + V_w \hat{j} \quad \text{and} \quad \vec{V}' = V(-\cos \beta \hat{i} + \sin \beta \hat{j}). \quad (2-13)(2-14)$$

$\beta$ , the angle between  $\vec{V}'$  and the negative y axis, can be defined in terms of the setting angle, the velocity of the wind, and the linear velocity of the blade:

$$\beta = (\theta - \alpha) = \left( 2\theta + \phi - \frac{\pi}{2} \right) \quad \text{where} \quad \phi = \tan^{-1} \frac{V_b}{V_w}. \quad (2-15)(2-16)$$

Using trigonometric identities Equation 2-14 for  $\vec{V}'$  simplifies to

$$\vec{V}' = V(-\sin(2\theta + \phi) \hat{i} - \cos(2\theta + \phi) \hat{j}). \quad (2-17)$$

Finding the difference between the y components of  $\vec{V}_r$  and  $\vec{V}'$  gives  $\Delta V_y$ :

$$\Delta V_y = V_y' - V_{ry} = -V \cos(2\theta + \phi) - V_w. \quad (2-18)$$

The goal, however is to describe the recoil force imparted to the blade. From Newton's third law, the momentum change per unit time delivered to wind by the blade results in a continuous reaction force on the blade. This in turn results in a continuous torque on the generator. The direction of the reaction force of the blade is opposite the velocity change of the wind, or

$$\Delta V_y = V \cos(2\theta + \phi) + V_w. \quad (2-19)$$

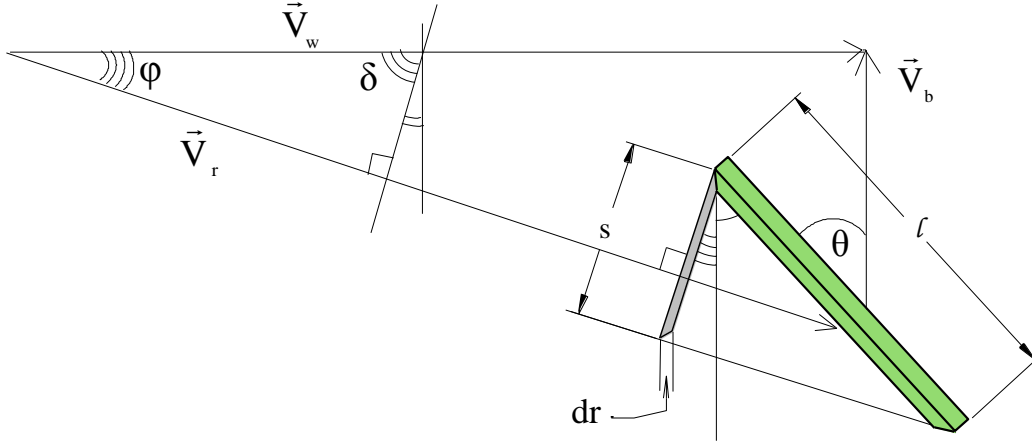
## 2.4 Finding the effective area

The effective area is the area of the blade perpendicular to the wind's relative velocity. As seen in Figure 14,  $dA_{\text{eff}}$  has length  $s$  and width  $dr$ . The length  $s$ , in terms of the blade parameters, is

$$s = l \cos(\lambda + \theta) \quad (2-20)$$

where  $l$  is the actual width, or chord, of the blade. Using geometry, it is seen that  $\lambda$  is equal to  $\phi$ . The effective area is thus

$$dA_{\text{eff}} = s dr = l \cos(\phi + \theta) dr \quad (2-21)$$



**Figure 14.** Mass flow rate. The small mass of air with relative velocity  $\vec{V}_r$  passes through the effective area of the blade. This area is perpendicular to the air's relative velocity, and has a width of  $dr$  and a height of  $s$ .

## 2.5 Finding power

Since the blades are rotating with angular velocity  $\omega$ ,  $V_b$  is  $r\omega$ . From Equation 2-11 therefore, with  $V_b = r\omega$  and substituting in  $\Delta V_y$  from Equation 2-19 and  $dA_{\text{eff}}$  from Equation 2-21, the power transferred from the small air mass to the small blade segment of width  $dr$  is

$$dP = \omega pr V l \cos(\phi + \theta) (V \sin(2\theta + \phi) + r\omega) dr \quad (2-22)$$

The power transferred to the entire blade is found by integrating the power transferred to each small segment over the whole range of the blade's radius (from  $r = a$  at the root to  $r = b$  at the tip):

$$P = \int_a^b \omega pr V l \cos(\phi + \theta) (V \sin(2\theta + \phi) + r\omega) dr \quad (2-23)$$

where

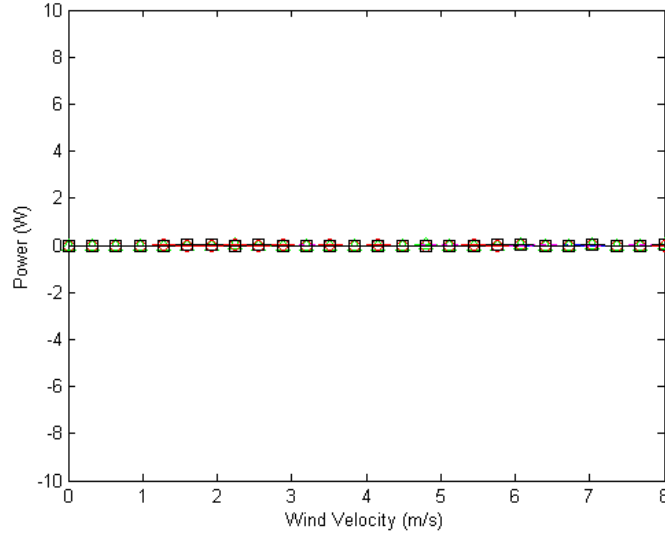
$$V = \sqrt{(\omega r)^2 + V_w^2} \quad \text{and} \quad \phi = \tan^{-1} \frac{\omega r}{V_w} \quad (2-24)$$

## 2.6 MATLAB Integration

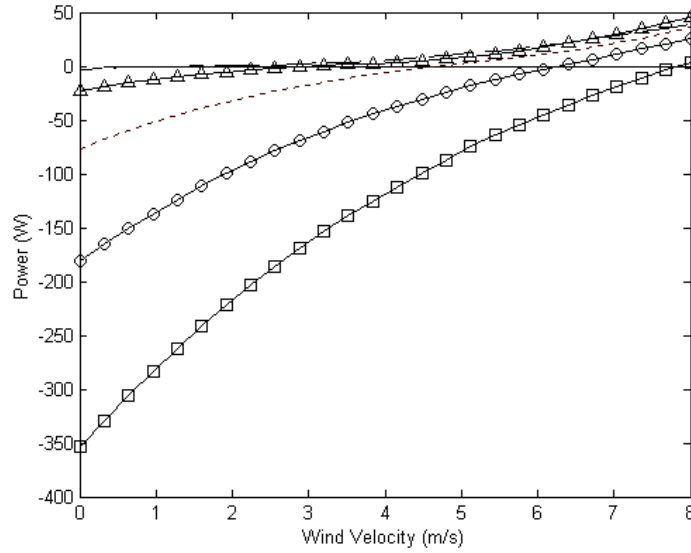
The power integral in Equation 2-23 cannot easily be solved analytically, so MATLAB was used to solve it numerically. MATLAB [16] stands for “matrix laboratory”, and is a matrix-vector-oriented programming language and software system for technical computing and numerical analysis distributed by MathWorks, Inc. One of the numerical integrating procedures in MATLAB is the “quad” function that uses Simpson’s rule. The “quad” function was used to evaluate the power integral, Equation 2-23, with given values for  $\theta$ ,  $l$ ,  $\rho$ ,  $\omega$ , and  $V_w$ . This program is in Appendix A.

An initial test was done to see if the power integral returned appropriate values for extreme cases of parameter and wind values. If the setting angle is  $0^\circ$ , power should be zero because the effective area includes all the blade area. There is no change in the y component of the wind velocity, so no power is transferred to the blade. If the wind velocity is 0 m/s, then the power output of the blades should either be zero or negative, depending on the angular velocity. In this case the blade is acting like a fan, and the higher the blade’s angular velocity, the more power it transfers to the air. When the setting angle is 90 degrees along the entire blade, the power output should not depend on the wind velocity. Also, similar to the case when the wind velocity was zero, power output should be zero or negative depending on  $\omega$  because the rotating blades have no effective area and thus are pushing the air around. As seen in Figure 15, Figure 16, and Figure 17, the power equation meets all of these expected conditions.

The values used for constants were air density,  $\rho = 1.21 \text{ kg/m}^3$ ; blade length,  $L = 1.5 \text{ m}$ ; and integration limits,  $a = 0.1 \text{ m}$ ,  $b = 1.5 \text{ m}$ . The lower limit of integration does not go to  $r = 0$  at the center, because there will be a central hub to which the blades will be mounted and which should not be considered as part of the blade in the power equation. Also, at this time the power transferred to just one blade is considered.

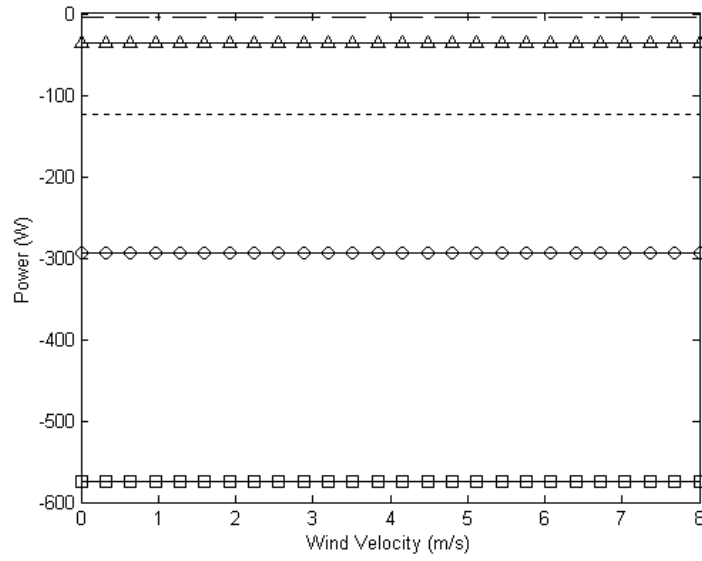


**Figure 15.** Plot of power output as a function of wind velocity for a setting angle of  $0^\circ$ . Power output is zero since there is no momentum transfer to the blade from the wind. The angular velocity was given several values,  $\omega = 1$  rad/s (solid line),  $\omega = 2$  rad/s (dashed line),  $\omega = 3$  rad/s (triangles),  $\omega = 3$  rad/s (dotted line),  $\omega = 4$  rad/s (diamonds), and  $\omega = 5$  rad/s (squares).



**Figure 16.** Plot of power output showing negative power when  $V_w = 0$  m/s. When the velocity of the wind is 0 m/s, the power output for each curve is negative because the rotor is moving with angular velocity  $\omega$  and thus requiring power to the air around. The angular velocity was given several values,  $\omega = 1$  rad/s (solid line),  $\omega = 2$  rad/s (dashed line),  $\omega = 3$  rad/s (triangles),  $\omega = 3$  rad/s (dotted line),  $\omega = 4$  rad/s (diamonds), and  $\omega = 5$  rad/s (squares).





**Figure 17.** Plot of power output as a function of wind velocity for a setting angle of  $90^\circ$ , so the blades have no effective area. As the angular velocity of the rotor increases, the blades are pushing more wind, and so the power output decreases because. The angular velocity was given several values,  $\omega = 1$  rad/s (solid line),  $\omega = 2$  rad/s (dashed line),  $\omega = 3$  rad/s (triangles),  $\omega = 3$  rad/s (dotted line),  $\omega = 4$  rad/s (diamonds), and  $\omega = 5$  rad/s (squares).

## Chapter 3

### MAXIMUM POWER SEARCH

After the initial test of the power integral for extreme cases, a search was made for the set of parameter values that gave the most power output at a particular wind velocity. Three different methods of searching were done, each successively more comprehensive.

#### 3.1 Modification of the power integral

During the maximum power search the original power integral, Equation 2-23, was modified and improved. In the original power integral  $\theta$ , the setting angle, and  $\omega$ , the angular velocity, are constant. These two parameters were allowed to vary along the radius and with wind velocity, respectively, to improve power output.

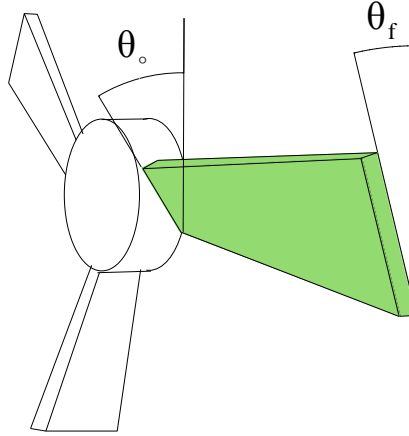
##### 3.1.1 $\theta$ dependence upon $r$

The tip of the blade moves much faster than the root of the blade as it rotates. With a constant setting angle along the entire radius, the tip might push the air rather than be pushed by the air because the blade may be moving too fast for the wind velocity to transfer positive power. This is not as important when  $\omega \leq 1$  rad/s because then the rotor is generally slower than the wind. As is seen in Figure 19 and Figure 20, however, this effect becomes important for high values of  $\omega$ . For the tip of the blade to contribute to the power output, the effective area needs to increase along  $r$  such that it is greatest at the tip, that is,  $\theta_o > \theta_r$ , as seen in Figure 18.

A simple exponential relationship between  $\theta$  and  $r$  was chosen:

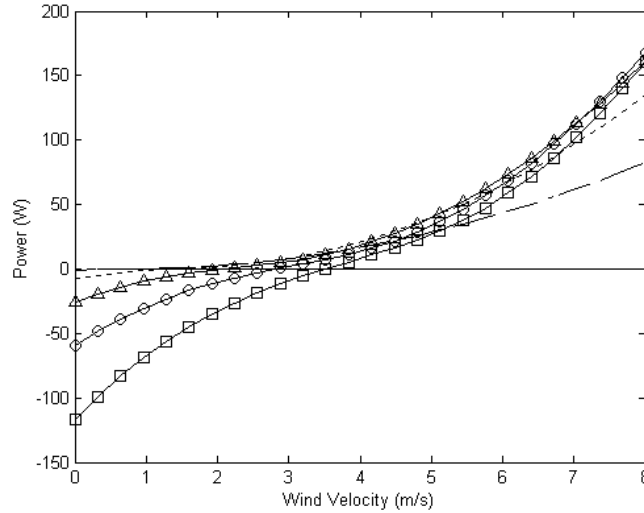
$$\theta = \theta_o e^{-3r/2L} \quad (3-1)$$

where  $\theta_o$  is the setting angle at the root and  $L$  is the total length of the blade.

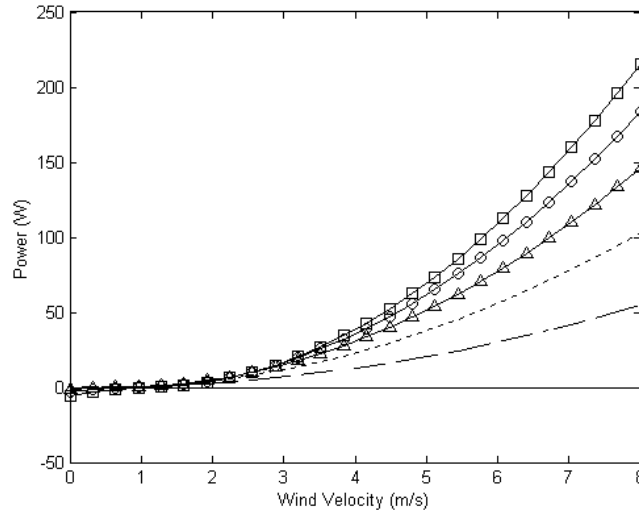


**Figure 18.** Setting  $\theta_f$  to be less than  $\theta_o$  increases the effective area at the tip.

Power curves generated from substituting Equation 3-1 into Equation 2-23 for  $\theta_o = 30^\circ$  are seen in Figure 20. Power curves generated from the original power integral are seen in Figure 19 for  $\theta = 30^\circ$  for comparison. Power output was 25-50 W higher for  $\omega = 3, 4$ , and 5, and lower for  $\omega = 1$  and 2 rad/s.



**Figure 19.** Power curves from equation 2-23, for  $\theta = 30^\circ$  and wind velocities of 1 to 8 m/s.  $\theta$  and  $\omega$  are constant. The solid line is for  $\omega = 0$  rad/s, the dashed line for  $\omega = 1$  rad/s, the dotted line for  $\omega = 2$  rad/s, the triangles for  $\omega = 3$  rad/s, the circles for  $\omega = 4$  rad/s, and the squares for  $\omega = 5$  rad/s.



**Figure 20.** Power curves with  $\theta = \theta_o e^{-3\pi/2L}$ ,  $\theta_o = 30^\circ$  and wind velocities of 1 to 8 m/s.  $\omega$  is constant. The solid line is for  $\omega = 0$  rad/s, the dashed line for  $\omega = 1$  rad/s, the dotted line for  $\omega = 2$  rad/s, the triangles for  $\omega = 3$  rad/s, the circles for  $\omega = 4$  rad/s, and the squares for  $\omega = 5$  rad/s.

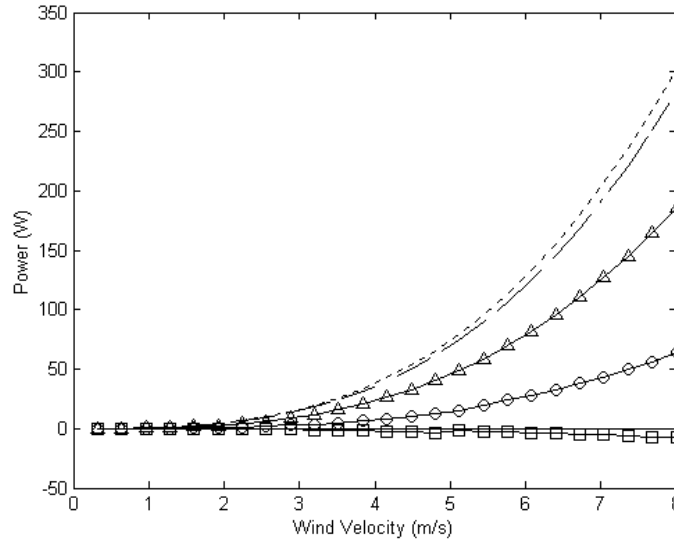
### 3.1.2 $\omega$ dependence upon $V_w$

When  $\omega$  is constant for all wind velocities, the power output is negative when the wind is slow enough that the blades are pushing the air rather than the wind pushing them. This is seen in Figure 16 and Figure 17. For the power output at small wind velocities to be positive and to generally improve power output at all wind velocities,  $\omega$  was allowed to vary with  $V_w$ . This would be accomplished by increasing or decreasing the electrical load on the generator and thus changing the back torque on the rotor from the generator.

The best relationship found from those considered was

$$\omega = \omega_o V_w$$

with  $\omega_o$  is a coefficient with units of radians per meter when  $\omega$  is in rad/s and  $V_w$  is in m/s. As seen in Figure 21, power output reaches up to about 300 W when  $\omega_o = 1$  and 2 rad/m.



**Figure 21.** Power curves for  $\omega = \omega_o V_w$  and  $\theta = \theta_o e^{-3v/2L}$  with  $\theta_o = 30^\circ$ . The solid line is for  $\omega_o = 0$  rad/m, the dashed line for  $\omega_o = 1$  rad/m, the dotted line for  $\omega_o = 2$  rad/m, the triangles for  $\omega_o = 3$  rad/m, the circles for  $\omega_o = 4$  rad/m, and the squares for  $\omega_o = 5$  rad/m.

### 3.2 MATLAB programming

Several search methods using MATLAB were used to look for the combination of parameter values that resulted in maximum power output. Equation 2-23 was searched to find the optimal value of each parameter by visual, comparative, and Simplex methods.

#### 3.2.1 Visual search

Families of power curves were generated and plotted on the same graph for comparison. Each family of power curves had the same set of parameter values except for the angular velocity. Thus for a given value of  $\theta_o$ , the best matching  $\omega_o$  value was found, at least compared to the other four values of  $\omega_o$  plotted. Figure 22 shows six of these graphs that include power curves for a range of  $\omega_o$  and  $\theta_o$  values.

Often the power curve that gave the highest power output depended on the wind velocity. In Figure 22C, for example,  $\omega_o = 3$  rad/m seems best with a wind velocity of about 3 m/s, but  $\omega_o = 4$  rad/m seems best with a wind velocity of 6 m/s and above. For this reason the

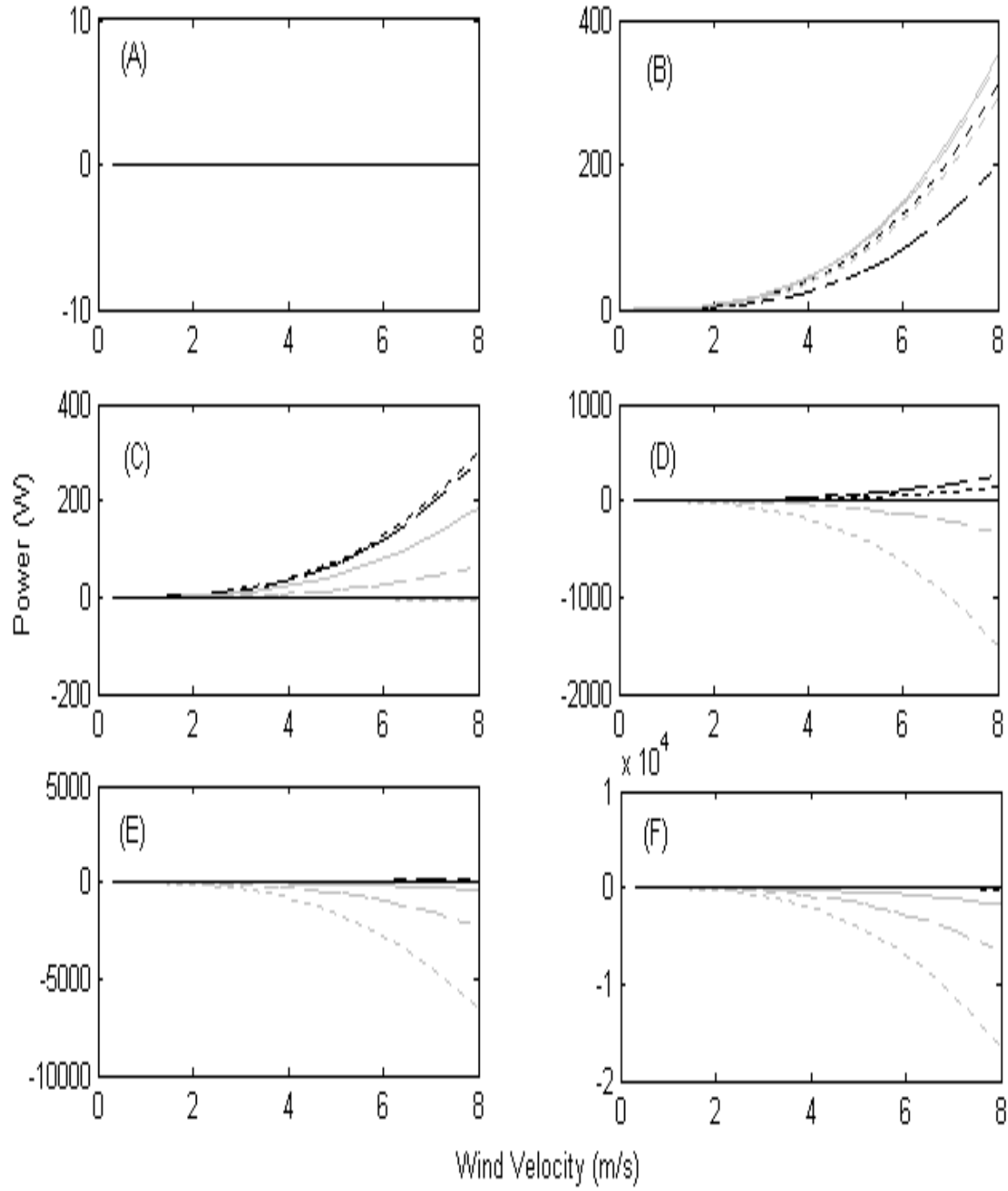
power output for a wind velocity of 4 or 5 m/s (the average wind velocity around Houghton) was especially considered when looking for the ideal set of parameters. By this method of searching, the optimal values were  $\omega_o = 2$  rad/m and  $\theta_o = 27^\circ$ . The power output with these values is seen in Figure 22. The program for the visual search is in Appendix C.

One of the problems with this type of search, however, is that only a few sets of values may be compared at one time. Also, this search focused around parameter values that are typical of current turbines. The following comparative and simplex search methods attempt to avoid these problems.

### 3.2.2 Comparative search

The comparative search is a simple search algorithm that evaluated the Equation 2-23 over a set range of  $\omega_o$  and  $\theta_o$ . Given a range over which each parameter and the number of “steps” to take within that range, the program evaluated Equation 2-25 at each set of “steps” and compared the results. The set of parameter values that produced the largest power output was determined and returned, along with the power output. This program is in Appendix D. When given wide ranges of  $\omega_o$  (1-100 rad/m, step of 1 rad/m) and  $\theta_o$  (1-45°, step of about 0.5294°), the search returned a maximum power value of 154 W with an optimal set of  $\omega_o = 56$  rad/m and  $\theta_o = 1.0588^\circ$  for a wind velocity of 6 m/s.

The problem with this search was that every time the range and step of the values for  $\omega_o$  and  $\theta_o$  changed, so did the result since it was limited to return only values that were stepped through. For example, if the range was 0 to 10 and the step was 2, the returned optimal value could only be 0, 2, 4, 6, 8, 10. However, if the step is 1.5, the returned value has to be 0, 1.5, 3, 4.5, 6, 7.5, or 9. Also, the search was bound by whatever ranges the program was given. Regardless of how wide the range and how small the step size, the search is still limited to returning a set of values that equal one of the step values.



**Figure 22.** Families of power curves. In subplot A,  $\theta_o = 0^\circ$ , in B,  $\theta_o = 18^\circ$ , in C,  $\theta_o = 36^\circ$ , in D,  $\theta_o = 54^\circ$ , in E,  $\theta_o = 72^\circ$ , and in F,  $\theta_o = 90^\circ$ . In each subplot the solid black line is the power curve for  $\omega_o = 0$  rad/m, the dashed black line for  $\omega_o = 1$  rad/m, the dotted black line for  $\omega_o = 2$  rad/m, the solid gray line for  $\omega_o = 3$  rad/m, the dashed gray line for  $\omega_o = 4$  rad/m, and the dotted gray line for  $\omega_o = 5$  rad/m.

### 3.2.3 Simplex search

The “fminsearch” function within MATLAB uses a simplex search to minimize a function of multiple variables. This method was used to search for the optimal set of values for  $\omega_o$  and  $\theta_o$  by negating the whole power equation and searching for the “minimum” value of power calculated. Given a starting point, the function did a non-gradient based search for a local minimum. The simplex program is in the Appendix E.

With a starting point of  $\omega_o = 0$  and  $\theta_o = 0$ , the simplex search returned a maximum power of 156.6022 W with an optimal, and highly unrealistic, set of  $\omega_o = 1335$  rad/m and  $\theta_o = 0.0441^\circ$  for a wind velocity of 6 m/s. This is a problem because the limits of blade strength prevent the rotor from moving this fast.



## *Chapter 4*

### DISCUSSION AND CONCLUSION

The results from the MATLAB searches are a good starting point for this project, and should be considered as preliminary work. The results from the two most comprehensive searches, the comparative and Simplex methods, are currently impractical for actual turbine design. For example, at  $V_w = 6$  m/s, the Simplex search returned an angular velocity of  $1335 V_w$  rad/s. However, blade fatigue and stress limit the rotor's angular velocity, and thus it is highly unrealistic to set a rotor to an angular velocity of this magnitude.

The results of these searches have made it obvious that several important factors have not been included in the power integral. Frictional forces between the rotor and the shaft, and the shaft and the generator need to be considered as well as drag forces. These may limit the search to practical limits. An analysis of airfoil blade shapes should be done. An airfoil blade shape will improve power output by decreasing the air pressure on the top of the blade, and thus pushing the blade up. The effect of varying blade width may yet be considered. When the tip of the blade is going much faster than the wind, it may be advantageous for the tip to be narrower than the root of the blade. That way the tip will not have as much area to push the wind, so that it will lose less energy.

## APPENDIX- MATLAB PROGRAMS

All of the programs described in the Appendix were written in and run from MATLAB M-files. There are two types of M-files. Script files are those that contain a sequence of commands, and function files typically contain a shorter sequence and are particularly useful for repeating a command many times [16]. In each case, the entire program includes a script file that calls several function files. The programs are so divided because it allows the equation to be found and changed more easily.

Each program integrates over the blade radius ( $r$ ) from point  $a$ , near the hub of the blade, to  $b$ , at the tip. Program lines have a solid line on the left margin, and comments within the programs are in **bold**.

## Appendix A. Basic program

The “basic program” numerically integrates the power integral (Equation 2-23) with the “quad” function and plots a family of power curves. Each power curve has the same set of parameters except  $\omega$ . “Basicpowerchangew” is the script file that calls the “basicquad” function file. The “basicquad” file includes the numerical integration function “quad”, which calls the “basicfunction” file that contains the power equation.

Variables used in this program are the blade radius ( $r$ ), the wind velocity ( $v$ ), the density of the wind ( $\rho$ ), the total length of the blade ( $L$ ), the setting angle ( $q$ ), and the angular velocity ( $\omega$ ). The values or range of values of each variable may be set in script file.

### 1. “basicfunction”

This is a function file that contained the power equation (Equation 2-23) to be evaluated. The list of variables to the right of “fpower” is in the order that the values are passed to the function. All variables that are passed to the function as an array, such as  $r$ , since it is the value being integrated over, require a dot (.) after them before they are squared or multiplied by a non-array value.

```
function y = fpower(r, v, p, L, q, w)
y=w*p*L*abs(r.*(((r.^2)*w^2+v^2).^(1/2)).*cos(q+atan(r*w*v^(1))))).*(((r.^2)*w^2+v^2)...
^(1/2)).*sin(2*q+atan(r*w*v^(-1)))-r*w)
```

### 2. “basicquad”

This function file includes the “quad” function that numerically integrates the power equation using Simpson’s rule. Variables  $a$  and  $b$  are the bounds of integration.

```
function y = quadpower(v, a, b, p, L, q, w)
y = quad('basicfunction', a, b, .1, 1, v, p, L, q, w);
```

### 3. “basicpowerchangew”

This script file creates and plots the family of power curves for a given range of wind velocity and given values of blade length and setting angle. Each curve has a different angular velocity.

Figure This creates the figure into which the curves are plotted

`m = 8;` This is the maximum wind velocity (m/s) graphed

`X = 0:m;` This defines the x-axis on the plot

`L = 1.5;` total length of blade

`s = 0;` initial value of  $\omega$  (rad/s)

`l = 5;` final value of  $\omega$

`n = (l-s)/5;` the step of  $\omega$

`theta = pi/4;` value of theta plotted (rad)

This is a for loop to plot for each value of  $\omega$ :

`for i = s:n:l`

`[X,Y]=fplot('basicquad', [0 m], .1, 25, '-',1, L, 1.21, L, theta, i)`

This part switches through each value of  $\omega$ , plots a different color for each value, and assigns each value of  $\omega$  to a letter for the legend display

`switch i`

`case s`

`plot(X,Y,'k');`

`a = i;`

`case n + s`

`plot(X,Y,'b');`

`b = i;`

`case 2*n + s`

`plot(X,Y,'g');`

`c = i;`

`case 3*n + s`

`plot(X,Y,'m');`

`d = i;`

`case 4*n + s`

`plot(X,Y,'r');`

`e = i;`

`case 5*n + s`

`plot(X,Y,'y');`

`f = i;`

`otherwise`

`plot(X,Y,'c');`

`g = i;`

`end`

This legend displays each value of  $\omega$  stepped through, and is put in the upper left corner.

`legend(sprintf('w = %1.1f rad/s',a),sprintf('w = %1.1f rad/s',b), sprintf('w = %...`

`1.1f rad/s',c),sprintf('w = %1.1f rad/s',d),sprintf('w = %1.1f rad/s',e),sprintf('w = ...`

```

%1.1f rad/s',f), 2);

    hold on
end

xlabel(' Wind Velocity (m/s) ');
ylabel(' Power (W) ');
title('Basic function: all parameters are constant along r, theta = ')
hold off;

```

## Appendix B. Modified equations

These equations were in the “basicfunction” file. In the first one  $\theta = \theta_{\circ} e^{-3r/2L}$  and  $\omega$  is constant, so  $q = \theta_{\circ}$ . In the second one,  $\theta = \theta_{\circ} e^{-3r/2L}$  and  $\omega = \omega_{\circ} V_w$ , so  $q = \theta_{\circ}$  and  $w = \omega_{\circ}$ .

1. Modified Equation with  $\theta = \theta_{\circ} e^{-3r/2L}$

$$y = w * p * L * \text{abs}(r * (((r.^2) * w^2 + v^2).^{(1/2)}). * \cos(q * \exp((-r*3)/(2*L)) + \text{atan}(r*w*v^{...} \\ (-1))))). * (((r.^2) * w^2 + v^2).^{(1/2)}). * \sin(2*q * \exp((-r*3)/(2*L)) + \text{atan}(r*w*v^{(-1)}) - r*w);$$

2. Modified Equation with  $\theta = \theta_{\circ} e^{-3r/2L}$  and  $\omega = \omega_{\circ} V_w$

$$y = v * w * p * L * \text{abs}(r * (((r.^2) * (v*w)^2 + v^2).^{(1/2)}). * \cos(q * \exp(... \\ (-r*3)/(2*L)) + \text{atan}(r*v*w*v^{(-1)}))) * (((r.^2) * (v*w)^2 + v^2).^{(1/2)}). * \sin(2*q * \exp(... \\ (-r*3)/(2*L)) + \text{atan}(r*v*w*v^{(-1)}) - r*v*w);$$

## Appendix C. Visual search

This program used the “basicquad” function file described in Appendix A but with the modified function 2 in Appendix B. A nested for loop creates a set of six plots, each for a different value of  $\theta_o$ . Each plot contained a family of power curves with 5 different values of  $\omega$ .

“searchdoubleforloop”

m = 8; **maximum wind velocity (m/s) graphed**

X = 0:m

L = 1.5; **total length of blade**

initialj = 0; **initial values of  $\theta_o$  (rad)**

finalj = pi/2; **final value of  $\theta_o$**

n = (finalj-initialj)/5; **the step of  $\theta_o$**

figure

num=0;

**The outer for loop to plot for each value of  $\theta_o$**

for j = initialj:n:finalj

num=(num+1);

subplot(3,2,num)

initialg = 0 ; **initial value of  $\omega_o$**

finalg = 5; **final value of  $\omega_o$**

N = (finalg-initialg)/5; **the step of  $\omega_o$**

h = j\*(180/pi);

**inner for loop to plot for each value of  $\omega_o$**

for g = initialg:N:finalg

if h <= 0

axis([0 m -10 10])

end

[X,Y]=fplot('basicquad', [0 m], .1, 25, '-',1, L, 1.21, L, j, g); **plotting function**

**This part switches through each value of  $\omega_o$ , plots a different color for each value, and assigns each value of  $\omega_o$  to a letter for the legend display**

switch g

case initialg

plot(X,Y,'k');

a = g;

case N + initialg

plot(X,Y,'b');

b = g;

case 2\*N + initialg

plot(X,Y,'g');

```

        c = g;
    case 3*N + initialg
        plot(X,Y,'r');
        d = g;
    case 4*N + initialg
        plot(X,Y,'c');
        e = g;
    case 5*N + initialg
        plot(X,Y,'m');
        f = g;
    otherwise
        plot(X,Y,'y');
    end
    hold on
end

xlabel(' Wind Velocity (m/s) ');
ylabel(' Power (W) ');
title(sprintf('q = %1.1f deg.', h))
hold off;
end

```

**This legend displays each value of  $\omega$  stepped through, and is put in the upper left corner**

```

legend(sprintf('w = %1.1f',a),sprintf('w = %1.1f',b), sprintf('w = %1.1f',c),sprintf('w = 
1.1f',d), sprintf('w = %1.1f',e),sprintf('w = %1.1f',f), 2);

```



## Appendix E. Comparative search

The comparative search is a simple algorithm that evaluates the passed equation a given number of times over a given range of values, then returns the set of values that returned the maximum power.

“searchmax”

```
v = 6;    maximum wind velocity (m/s) graphed
initialv = 4;
numv = 1;    step of wind velocity
initialw = 0;    initial value of  $\omega$  (rad/m)
finalw = 100;    final value of  $\omega$ 
stepw = 100;
bignumw = (finalw-initialw)/stepw;    step size of  $\omega$ 
initialq = 0;    initial values of  $\theta$  (rad)
finalq = pi/4;    final value of  $\theta$ 
stepq = 85
bignumq = (finalq-initialq)/stepq;    the step size of  $\theta$ 

l = .1;    starting radius of blade
L = 1.5;    length of blade
p = 1.21;    density of air
```

**These are variables that will be assigned the maximum value for power and corresponding values of wind velocity, angular velocity, and setting angle**

```
maxpower = 0;
wind = 0;
angvel = 0;
angle = 0;
```

**These are a series of for loops to find max power**

```
for w = initialw:bignumw:finalw
    for q = initialq:bignumq:finalq

y = quad('searchfunction', l, L, .1, 1, v, p, L, q, w);
%disp(sprintf(' %3.0f %3.0f %3.5f %10.6f', v,w,q,y))
```

**This part compares the result just calculated with the previous result. If the current value of power is greater, the current values of power, wind velocity, angular momentum, and setting angle are assigned to the appropriate variables. If the previous value of power was greater, the previous values remain assigned to the variables.**

```
if (y >= maxpower)
```

```

    maxpower = y;
    wind = v;
    angvel = w;
    angle = q;
else
    maxpower = maxpower;
    wind = wind;
    angvel = angvel;
    angle = angle;
end

    end
end

```

**these are conversions from radians to degrees for the setting angle**

```

angle = angle*(180/pi);
iangle = initialq*(180/pi);
fangle = finalq*(180/pi);

```

**These are a the values displayed at the end**

```

wind
initialw
finalw
stepw
iangle
fangle
stepq
maxpower
angvel
angle

```

## Appendix F. Simplex search

The Simplex search used the “fminsearch” function within MATLAB to find the optimal set of parameter values for a given wind speed, blade length, and air density. When multiple variables are being search over, “fminsearch” requires them to be defined as a single array. An array, x, was created and substituted into the modified power function with  $x(1) = \omega$  and  $x(2) = \theta_o$ .

### 1. “searchfunction”

**w = x(1) and q = x(2)**

function y = fpower(r, v, p, L, x)

```
y = -(v*x(1)).*p*L*abs(r.*(((r.^2)*(v*x(1)).^2+v.^2).^(1/2)).*cos(x(2)).exp(...  
(-r^3)/(2*L))+atan(r*(v*x(1))*v.^...  
(-1))))).*(((r.^2)*(v*x(1)).^2+v.^2).^(1/2)).*sin(2*x(2)).exp(...  
(-r^3)/(2*L))+atan(r*(v*x(1))*v.^(-1)))-r*(v*x(1)));
```

### 2. “searchquad”

function y = searchfminquad(x, a, b, v, p, L)

y = quad('searchfminfunction', a, b, .1, 1, v, p, L, x);

### 3. “searchfmin”

**w = x(1) and q = x(2)**

initialw = 0;      **initial value of  $\omega_o$  (rad/m)**

initialq = 0;      **initial values of  $\theta_o$  (rad)**

MaxIter = 1\*(10^4) **maximum number of iterations**

MaxFunEvals = 1\*(10^4) **maximum number of iterations**

a = .1;      **a and b are the integration bounds**

b = 1.5;

v = 6;      **wind velocity**

p = 1.21; **air density**

L = b;      **total blade length**

**This is the fminsearch function.**

[x, min] = fminsearch('searchfminquad', [initialq; initialw]...

,optimset('MaxFunEvals',MaxFunEvals, 'MaxIter', MaxIter), a, b, v, p, L);

**below are all the values to be displayed at the end**

MaxIter

MaxFunEvals

v

w = x(1)

q = (x(2))\*(180/pi)

maxpower = - min

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