A Test of the Validity of Inviscid Wall-Modeled LES

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Abstract

Computational expense is one of the main deterrents to more widespread use of large eddy simulations (LES). As such, it is important to reduce computational costs whenever possible. In this vein, it may be reasonable to assume that high Reynolds number flows with turbulent boundary layers are inviscid when using a wall model. This assumption relies on the grid being too coarse to resolve either the viscous length scales in the outer flow or those near walls. We are not aware of other studies that have suggested or examined the validity of this approach. The inviscid wall-modeled LES assumption is tested here for supersonic flow over a flat plate on three different grids. Inviscid and viscous results are compared to those of another wall-modeled LES as well as experimental data–the results appear promising. Furthermore, the inviscid assumption reduces simulation costs by about 25% and 39% for supersonic and subsonic flows, respectively, with the current LES application. Recommendations are presented as are future areas of research.

Methodology

In this study, the Favre-filtered compressible Navier-Stokes equations are solved in generalized coordinates. The utilized compressible LES solver [1] uses a sixth order compact finite difference scheme for spatial derivatives and the classical fourth order Runge-Kutta method for time integration. A sixth order spatial filter is used as an implicit subgrid scale (SGS) model. When applied to supersonic simulations, the present methodology uses locally-applied WENO-based characteristic filters and an adaptive version of the spatial filter for shock capturing. The present simulations utilize an equilibrium adiabatic wall model based on the log law and a digital filter-based approximate turbulent boundary condition at the inflow.

The outlined methods have worked well for turbulent boundary layer simulations and a variety of jet noise aeroacoustics cases (see ref. [1] for examples). However, computing the viscous terms of the Navier-Stokes equations in generalized coordinates requires many arithmetic computations. Furthermore, the viscous flux terms require spatial derivatives of the solution which necessitate communication between processes and increase the cost. These costs may be mitigated.

It is well known that the impact of viscosity is negligible in high Reynolds number flows except near walls. In wall-modeled LES, however, a coarse grid is used and the viscous near-wall region is not resolved. As a result, the resolved flowfield may be modeled as inviscid if a wall model is used to incorporate the effect of viscosity at the wall. Under this assumption, the viscous fluxes need not be calculated reducing simulation costs. Cost is further reduced by using an implicit SGS model that does not require spatial derivatives of the solution, minimizing interprocessor communication.

Simulation Details

To test the validity of the inviscid assumption, simulations are performed at a free stream Mach number of 1.69 and a Reynolds number of 469,000 based on the inlet boundary layer thickness, δ_{99i}. Flows are allowed to spatially develop until a Reynolds number based on momentum thickness, Re_θ, of 50,000 is reached. Using the transformation suggested by Kawai and Larsson, this is comparable to an incompressible flow at Re_θ = 36,000 [2]. These conditions match Kawai and Larsson's and are similar to those of the incompressible experiments from De Graaff and Eaton [3] at Re_θ = 31,000. All simulation results are transformed to equivalent incompressible data before comparisons are made.

Simulations are performed on three grids as detailed in Table 1. The size of each domain is 50 δ_{99i} in the streamwise direction, x, 15 δ_{99i} in the wall-normal direction, y, and 3 δ_{99i} in the spanwise direction, z. A maximum CFL number of 0.8 is also used for each. The effects of transients are minimized by simulating for 250 δ_{99i}/U_∞, where U_∞ is the velocity at the edge of the boundary layer. Statistics are collected for 250 δ_{99i}/U_∞.

Table 1. Simulation details for the described cases. The wall-normal grid spacing at the wall is Δy_w – the grid is stretched for $y > 2\ \delta_{99i}$. The instantaneous solution is fit to the log law at the wall model matching point for estimating the wall shear stress. The matching point is given in grid points off of the wall. Lastly, the cost reduction compares the inviscid vs. viscous simulation costs in core-hours.

Case	$\Delta x{=}\Delta z$ (δ_{99i})	Δy_w (δ_{99i})	Matching Point	Time step size (δ_{99i}/U_∞)	Total Grid Points (10^6)	Cores	(Grid Points per Core)$^{1/3}$	Cost Reduction
Coarse	0.1	0.05	2	0.042	1.5	48	31.5	24%
Intermediate	0.066	0.033	3	0.029	4.4	192	28.4	27%
Fine	0.042	0.0183	3	0.015	17	222	42.5	26%

Results

Figure 1 shows mean velocity profiles for each simulation compared with data from references [2] and [3]. Figure 2 shows Reynolds stresses for these same cases. Both figures show that accuracy generally improves when using finer grids. Figure 3 shows how the skin friction coefficient varies with the Reynolds number. The present data are bounded by the White [4] and Karman-Schoenherr (see [2]) correlations and are near the experimental data [3]. These figures demonstrate that the present results compare well with data from Kawai and Larsson and De Graaff and Eaton. The differences between viscous and inviscid results are likely small enough for many applications.

With regards to cost savings, Table 1 shows the cost reduction between viscous and inviscid cases. On average, the inviscid simulations ran 35% more quickly and cost 25% less. Subsonic simulations were also performed on the same grids and the inviscid cost reduction was about 39%. A greater reduction in cost is noted in subsonic simulations due to the use of shock-capturing methods in supersonic cases.

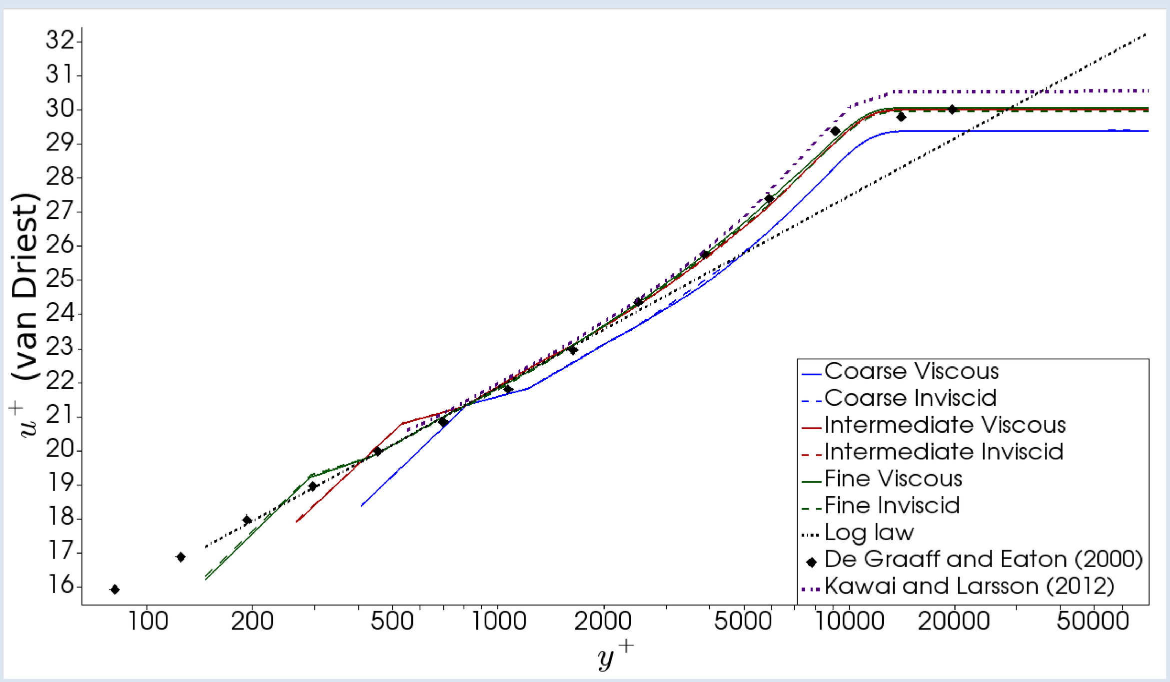

Figure 1. Van Driest transformed velocity vs. distance from the wall. u^+ represents the axial velocity nondimensionalized by the friction velocity. y^+ is the distance from the wall nondimensionalized with respect to the friction velocity as well as the density and dynamic viscosity at the wall.

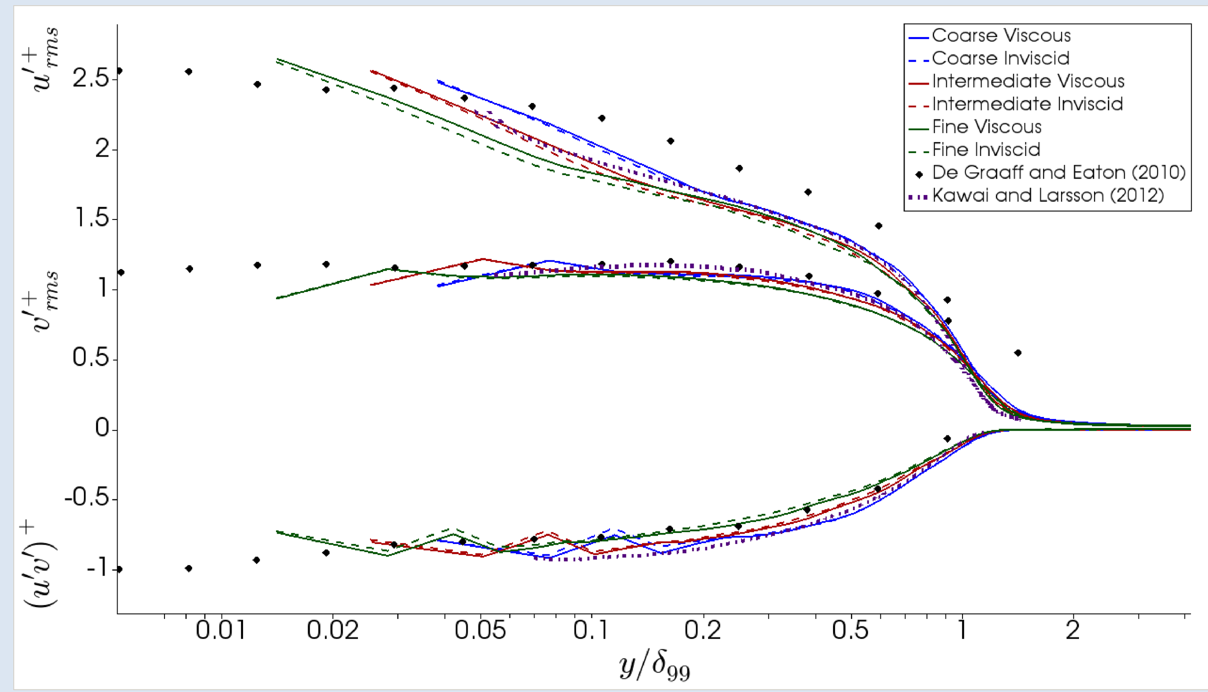

Figure 2. Reynolds stresses versus distance from the wall nondimensionalized by the local boundary layer thickness. The Reynolds stresses are nondimensionalized using the friction velocity. These results match those of Kawai and Larsson [2] well and are comparable with experimental data [3].

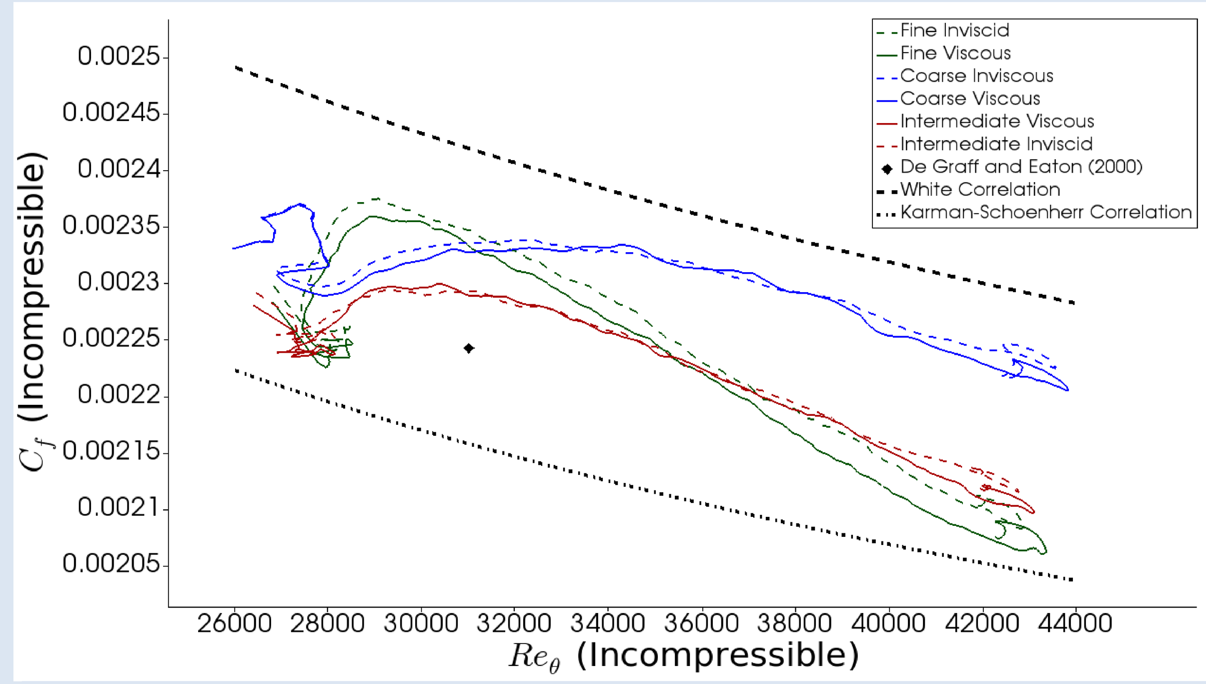

Figure 3. Shown here is a plot of skin friction coefficient, C_f, versus Reynolds number based on momentum thickness, Re_θ. Data on both axes have been scaled using the incompressible transformation used by Kawai and Larsson [2].

Conclusions

The results from the simulations using the inviscid assumption compare well with the computational results from Kawai and Larsson [2] as well as the experimental data from De Graaff and Eaton [3]. These comparisons suggest that making the inviscid assumption, while using wall-modeled LES, may not have a detrimental impact on the accuracy of the results. For flows at high Reynolds numbers, the viscosity has negligible effect on the outer flowfield and its impact in the near wall region is accounted for with the wall model. Notice that these conclusions are not dependent on the use of a specific wall model methodology. The main reason for making the inviscid assumption is to reduce simulation costs. The inviscid assumption improves efficiency by eliminating expenses associated with computing the viscous fluxes in the governing equations: this includes reductions in computation and in communication between processes. In the present application, simulation costs are reduced by about 25% and 39% for supersonic and subsonic flows, respectively. Future research could investigate the use of the inviscid wall-modeled LES assumption in more practical engineering applications.

References

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