A Study of Weak Magnetic Focusing

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I. Abstract

The small cyclotron at Houghton College loses at least 80% of the beam due to collisions with the dees and chamber walls. Weak magnetic focusing is being studied as a technique to reduce this problem by altering the magnetic field, which is currently nearly uniform, to create a greater radial magnetic field component which will create a restoring force to return ions to the central orbit plane. A computer model of the magnet and chamber is being to developed to design magnet shims that will give the most advantageous magnetic field shape for good focusing. A two dimensional cross section of the magnet has been modeled using Poisson Superfish, the results of which were used to track ions with the Simion 8.0 code. The model can be used to simulate various options for shim sizes and configurations the results of which will determine which shims will eventually be tested. Results of the computer model were compared with analytical results using a simplified model.

II. Theory

Cyclotron Basics

A cyclotron is type of particle accelerator. As seen in Fig. 1, cyclotrons have a central ion source between two oppositely charged hollow D-shaped electrodes called dees. The electric force on the ion from the dees' field causes the ion to move into one of the dees. Since there is a magnetic field through the entire chamber, the particle experiences a force orthogonal to both its velocity and the field (Eq. 1) and, as a result, follows a circular path. When the particle returns to the gap between the dees, the RF source has switched the polarity of the dees at a rate determined by the resonance frequency (Eq. 3) and the particle is once again accelerated across the gap.

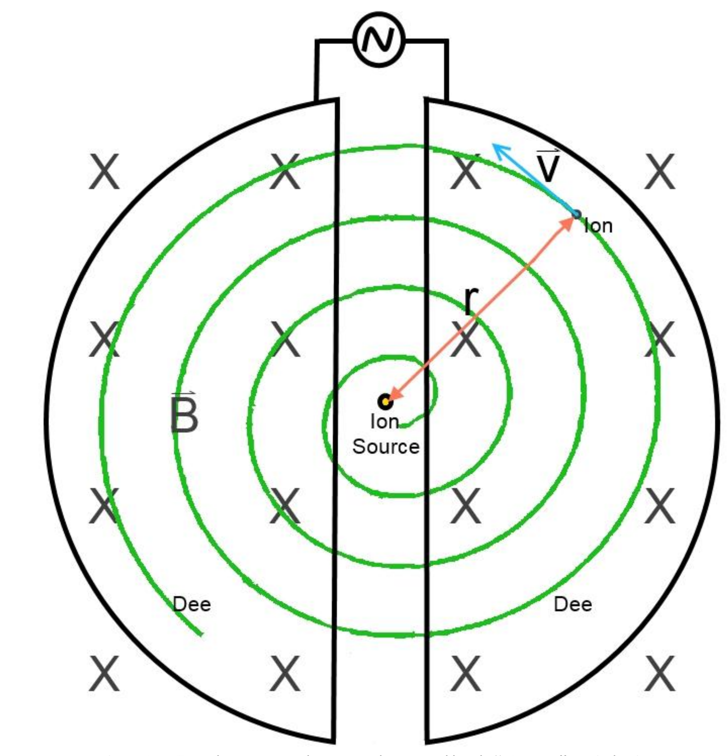

Fig. 1: Cyclotron electrodes called "Dees" with ion source at the center is placed into a magnetic field into the page. Ion path and velocity is indicated.

Cyclotron Equations

Using Newton's second law and the equation for centripetal acceleration, the radial component of the force on an ion of charge q (see Fig. 1) in a uniform magnetic field $\vec{B}$ is

$$F_r = -\frac{mv^2}{r} = -qvB \qquad (1)$$

which can be solved for the velocity of the ion

$$v = \frac{qBr}{m}. \qquad (2)$$

The orbit frequency can be found from the period T

$$f = \frac{1}{T} = \frac{v}{2\pi r} = \frac{qB}{2\pi m} \qquad (3)$$

which notably does not depend on radius. This is the mechanism which allows the cyclotron to operate at a set frequency. Furthermore, the energy of an ion at radius r is,

$$E = \frac{1}{2}mv^2 = \frac{q^2B^2r^2}{2m}. \qquad (4)$$

If the magnetic field designed for focusing goes like

$$B = B_0\left(\frac{r_0}{r}\right)^n, \qquad (5)$$

the axial equation of motion will be a harmonic oscillator

$$\frac{d}{dt}(m\dot{z}) = m\omega^2 nz \qquad (6)$$

yielding the frequency of axial oscillations

$$f_z = \sqrt{n}\, f_0. \qquad (7)$$

Focusing, Shims, and Vertical Oscillations

In general, particle accelerators require beam focusing to maximize the fraction of accelerated ions that reach the desired target. The magnetic field through a cyclotron chamber should (ideally) have azimuthal symmetry which means the two significant field components are axial and Radial (Fig. 2). The current Houghton cyclotron magnet creates a field through the chamber region with almost no radial component except near the edges due to fringing. The advantage of a field with an increased radial component is that, according to Lorentz's force law, a radial component of B results in a restoring force that is pointed back toward the central plane. One way of influencing the magnetic field shape is to insert shims – ferrous discs – above and below the vacuum chamber.

Focusing introduces harmonic oscillations in the axial direction as a result of the restoring force. The frequency of these harmonic oscillations depends on the field index, n, (Eq. 5) and resonance frequency (Eq. 3) as shown in Eq. 7. Typically, the field index increases with radius as does the frequency of axial oscillations.

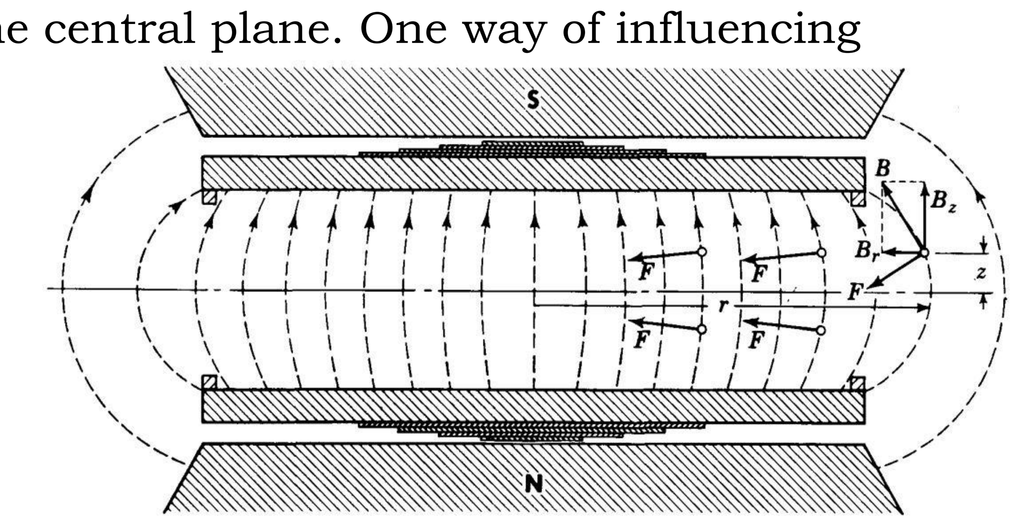

Fig. 2: Magnet pole tips shown above and below with shims between the poles and the chamber. Restoring force due to radial B always points towards the central plane. (Livingston and Blewett, *Particle Accelerators*, Fig. 6-7, pg 144).

III. Model

Poisson Superfish

Software Poisson Superfish (PSF) was used to model the cyclotron's geometry and calculate the magnetic field. The geometry can be adapted to predict the field generated by including magnet shims.

Simion

The magnetic field is imported into the Simion ion transport program which includes two electrodes shaped like the dees (Fig. 4 (a)). An ion initially at rest in the center spirals outwards due to the magnetic and electric field (Fig. 4 (b)). Dee voltage and resonance frequency can be set.

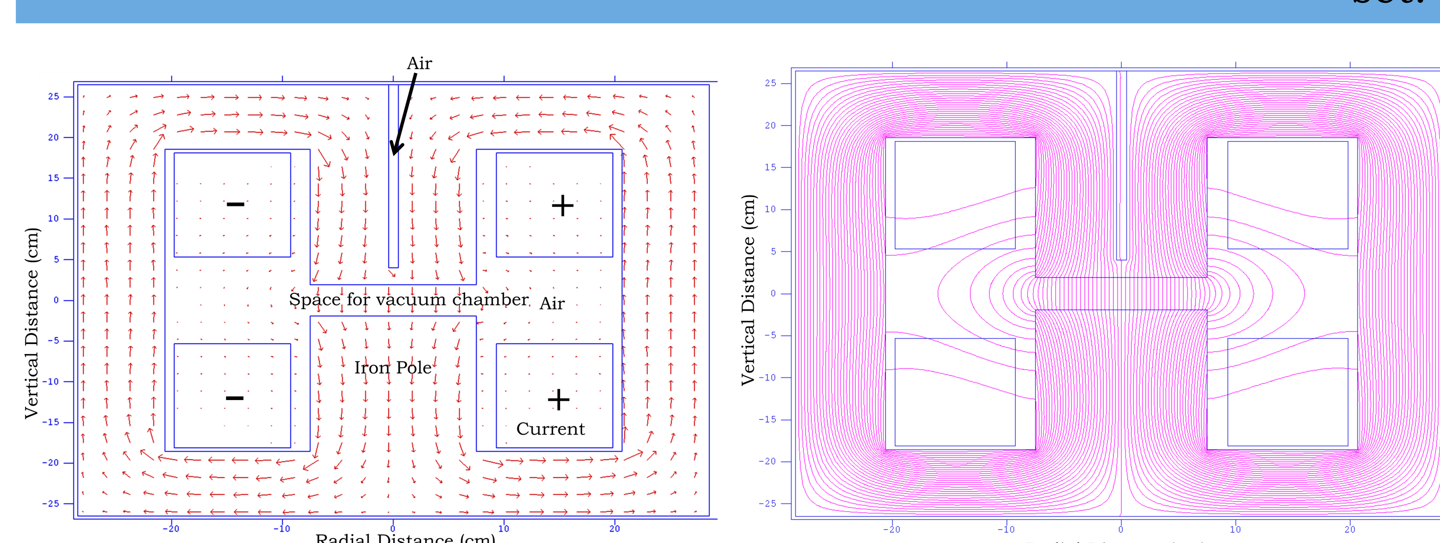

Fig. 3: Poisson Superfish model of the cyclotron magnet. Left: magnetic field vectors. Right: magnetic field contours.

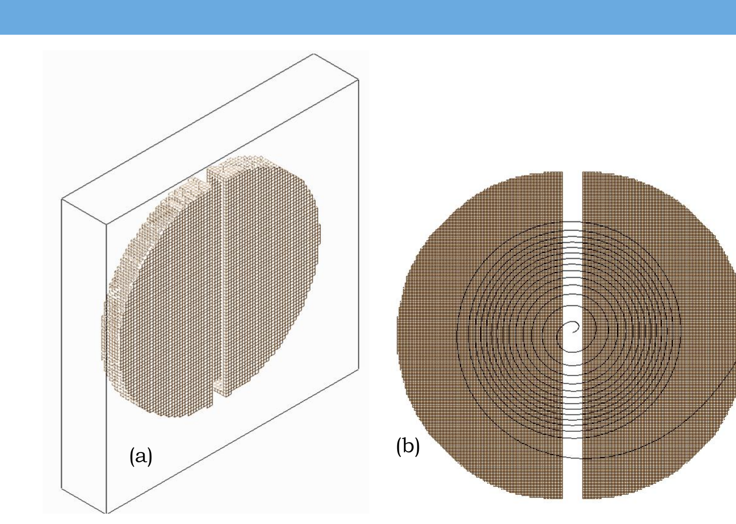

Fig. 4: (a) View of the dee geometry in Simion. These dees were designed by Dr. Timothy Koeth. (b) View of the particle path in Simion for the cyclotron at resonance frequency. This is for resonance frequency with dee voltage 13,000V.

The two dimensional model in Fig. 3 shows the cyclotron magnet cross section. The regions labeled positive or negative current are areas of current density in a particular direction representing coils.

IV. Results

Field Strength Comparison

Comparing the magnetic field strength predicted by PSF to measured values is a simple check to assure the model outputs a realistic solution. (Fig. 5).

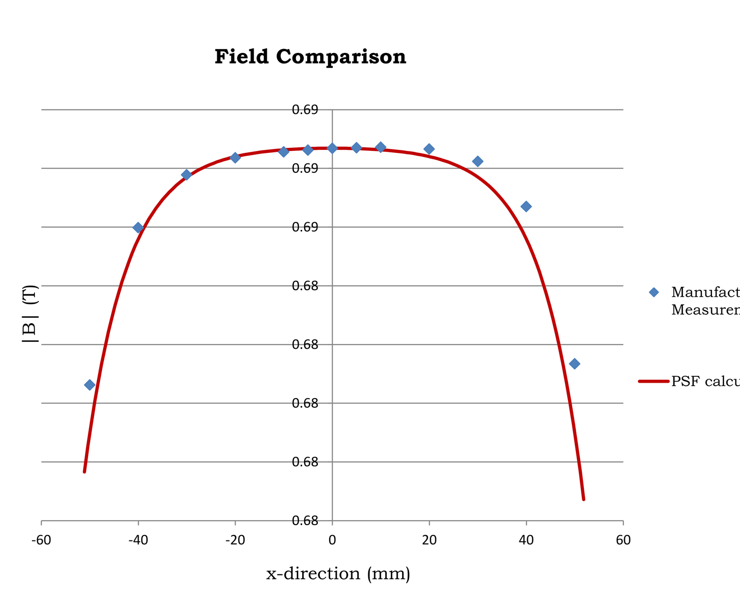

Fig. 5: Comparison between the magnetic field strength as measured by the manufacturer and the magnetic field strength found by PSF for the same geometry.

Path Focusing

Without focusing the ions path in the axial direction fluctuates over large distances (Fig. 10 top), but inserting the proper shim into the magnet should significantly decrease the size of fluctuations (Fig. 10 bottom).

Frequency Comparisons

An indication that the Simion model is giving realistic solutions is to compare the axial frequency of an ion from Simion with analytical calculations (Eq. 12) for the magnetic field used.

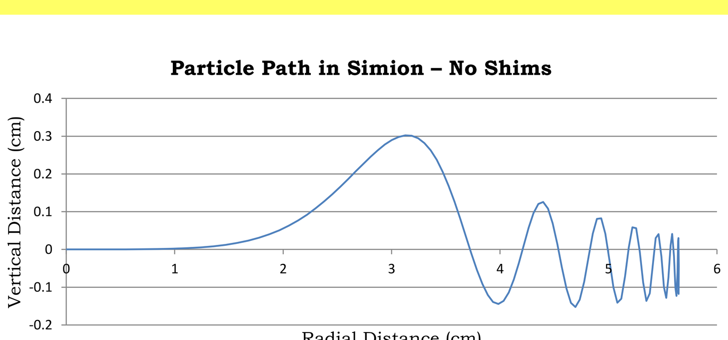

Fig. 6: Vertical oscillations for ω=121.4884 rad/μs, rfV = -3000V.

The time interval between successive zero crossings yields the approximate vertical frequency of an ion in Simion. During the first oscillation, the frequency was calculated using more points.

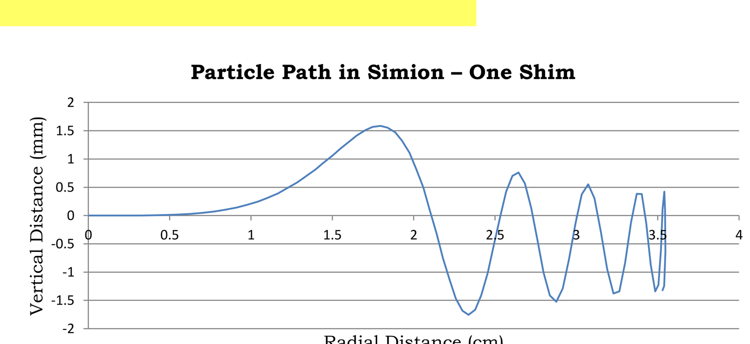

Fig. 8: Vertical oscillations for ω=121.4884 rad/μs, rfV = -3000V.

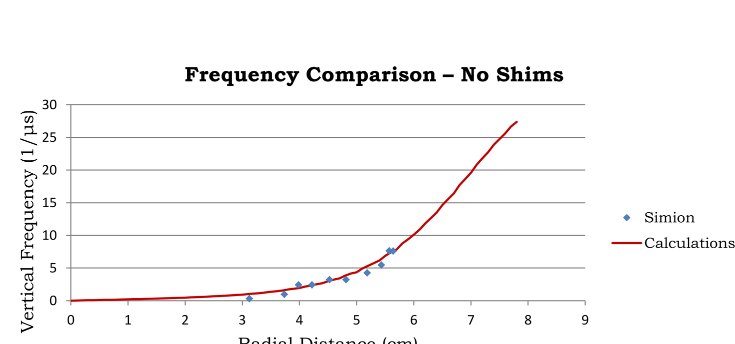

Fig. 7: Comparison between calculated vertical frequency based on the field at a particular radius, and the predicted frequency in the Simion simulation for ω=121.4884 rad/μs, rfV = -3000V.

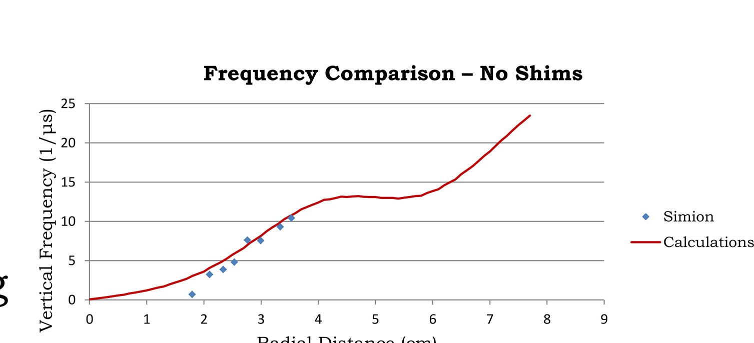

Fig. 9: Comparison between calculated vertical frequency based on the field at a particular radius, and the predicted frequency in the Simion simulation for ω=121.4884 rad/μs, rfV = -3000V.

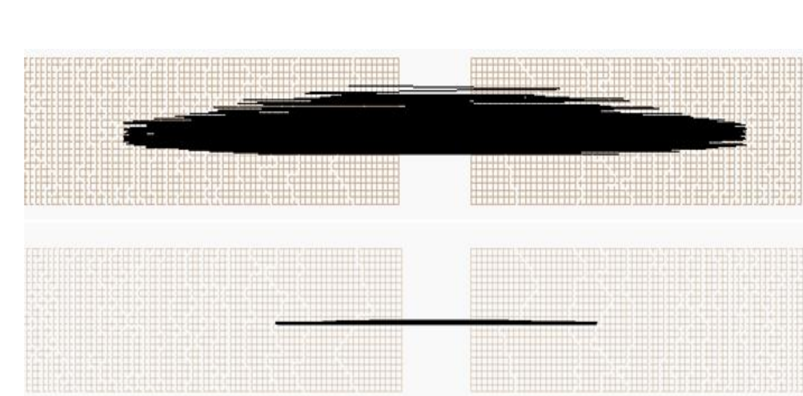

Fig. 10: Particle path in Simion as seen from a side view of the dees such that the axial direction is vertical. Top: No shims, ω=121.4884 rad/μs, rfV = -3000V. Bottom: One shims ω=121.4884 rad/μs, rfV = -3000V.

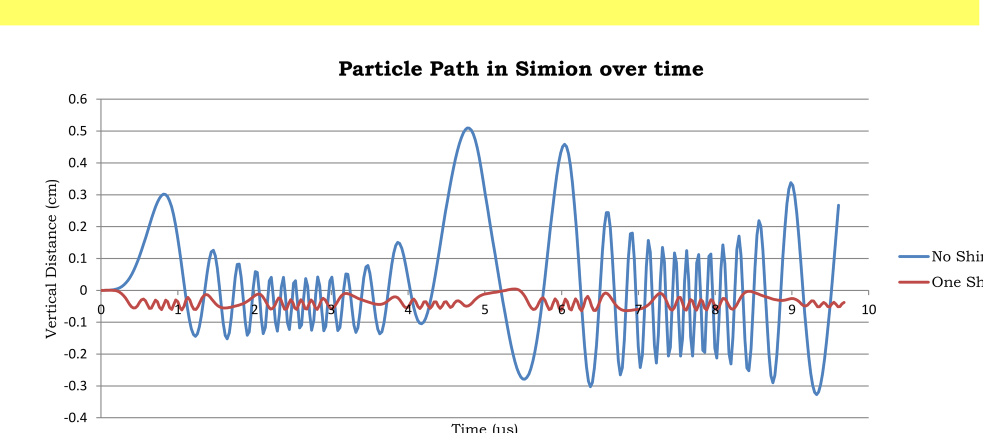

Fig. 11: Vertical distance of the ion with time. No shim: ω=121.4884 rad/μs, rfV = -3000V. One shim: ω=123.087998 rad/μs, rfV = -3000V.

V. Future Plans

The computer model of the cyclotron appears to function as expected. As a result, in the future design adaptations will be simulated before being implemented. The most impactful design upgrade will be shim inserts for field focusing which are currently being designed and will be tested as briefly summarized above. The resulting focused beam should significantly increase beam current.