A Wind Driven Power-Generating System: Preliminary Modeling and Analysis of Power Curves

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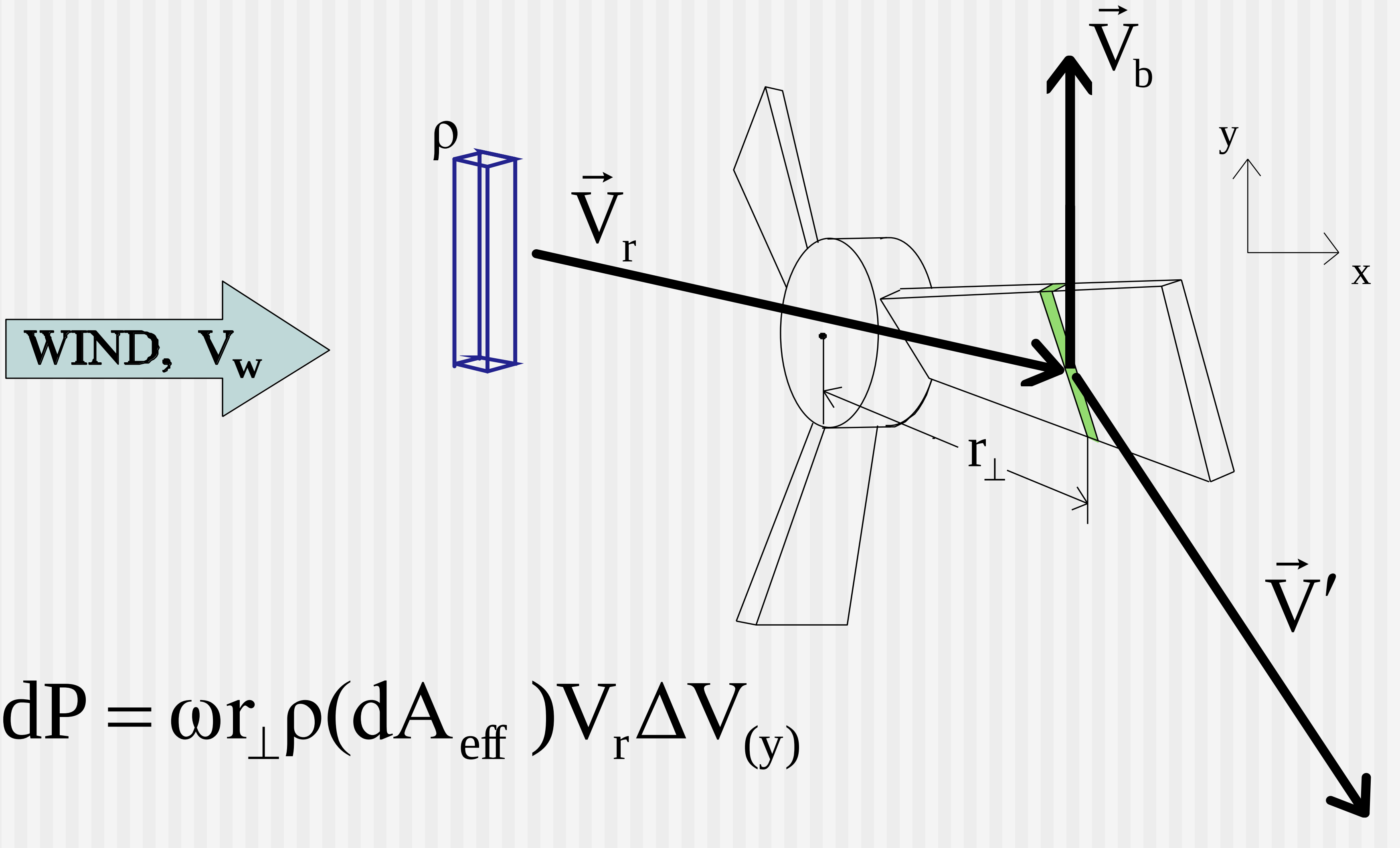
Outline

- Background and purpose
- Power Equation
- Preliminary evaluation

Background and Purpose

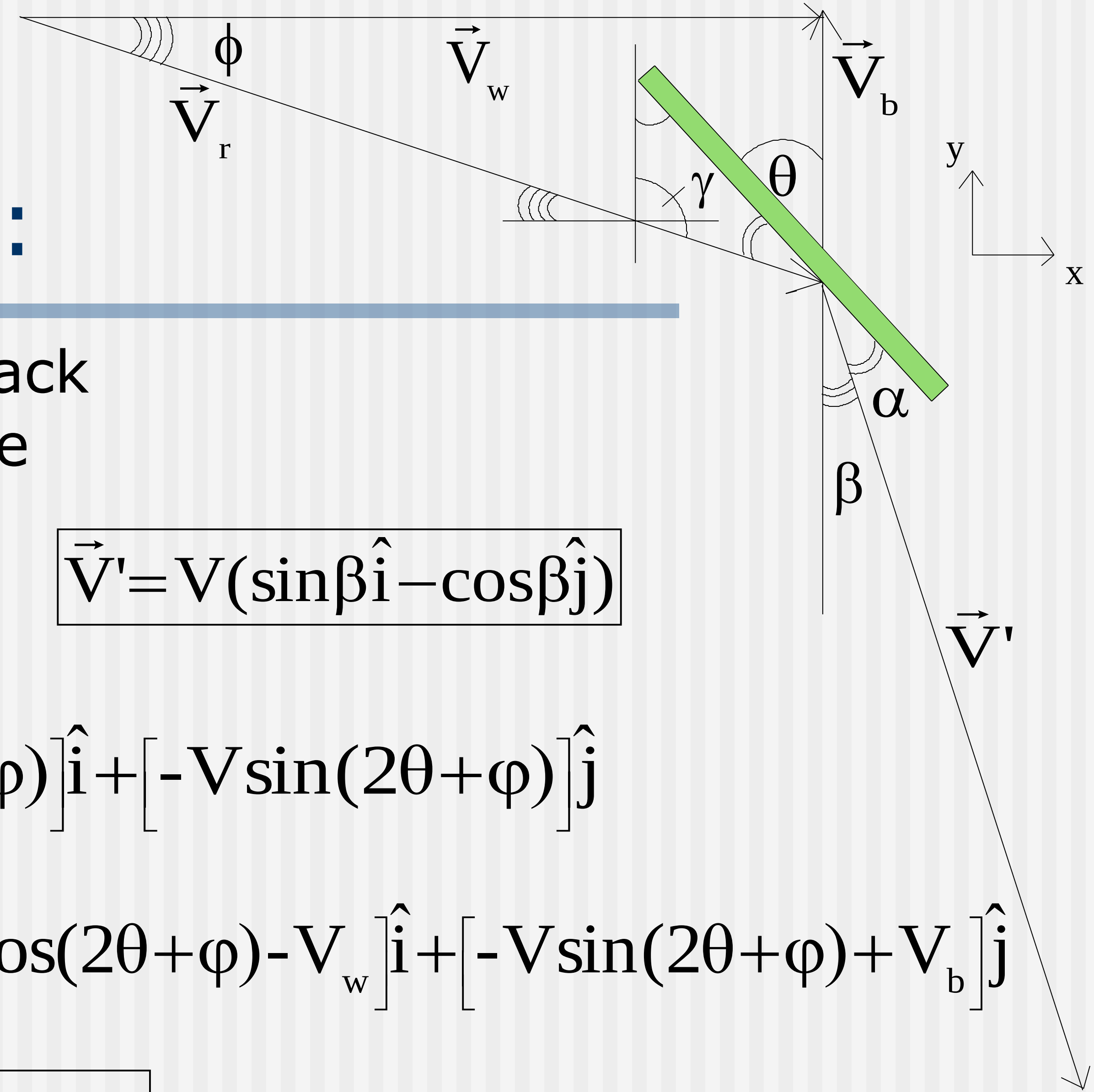
- Why wind power?
- Why our own design?

Derivation of power equation:



$$dP = \omega r_\perp \rho (dA_{eff}) V_r \Delta V_{(y)}$$

Finding ΔV:



α = angle of attack
θ = setting angle

$$\vec{V}_r = V_w \hat{i} - V_b \hat{j}$$

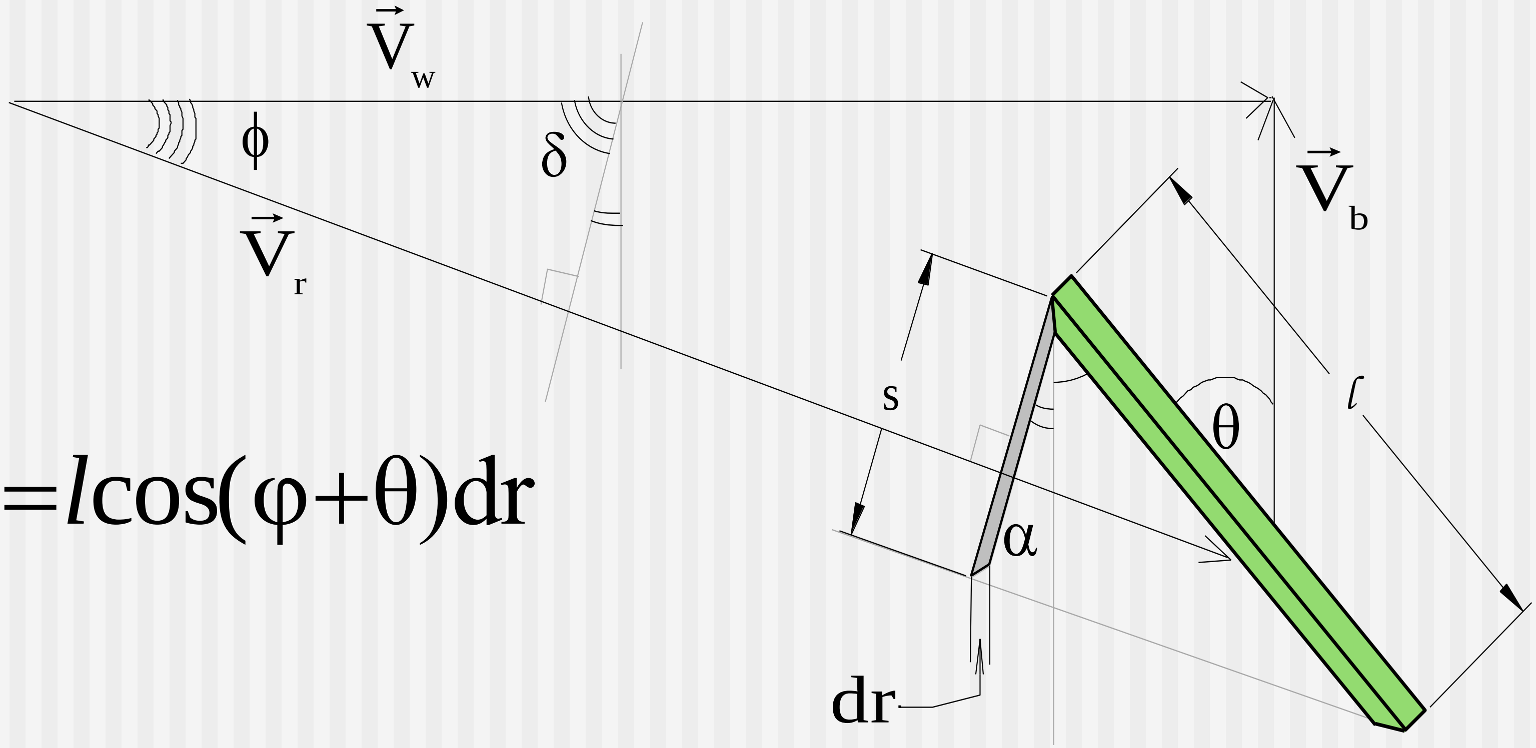
$$\vec{V}' = V(\sin\beta \hat{i} - \cos\beta \hat{j})$$

$$\vec{V}' = \left[-V\cos(2\theta+\varphi)\right]\hat{i} + \left[-V\sin(2\theta+\varphi)\right]\hat{j}$$

$$\Delta\vec{V} = \vec{V}' - \vec{V}_r = \left[-V\cos(2\theta+\varphi) - V_w\right]\hat{i} + \left[-V\sin(2\theta+\varphi) + V_b\right]\hat{j}$$

$$\Delta\vec{V}_y = -V\sin(2\theta+\varphi) + V_b$$

Finding A_{eff}:



$$dA_{eff} = l\cos(\varphi+\theta)dr$$

$$dP = \left(r\omega\left[\rho\left(l\cos\left(\varphi+\theta\right)\right)V\right]\left(V\sin\left(2\theta+\varphi\right)-\left(V_b\right)\right)\right)dr$$

Finding Power:

$$P = \int_a^b (r\omega)\left[\rho\left(l\cos\left(\varphi+\theta\right)\right)V\right]\left(V\sin\left(2\theta+\varphi\right)-(r\omega)\right)dr$$

with $\varphi = \tan^{-1}\frac{r\omega}{V_w}$ and $V = \sqrt{(r\omega)^2 + V_w^2}$

$$P = \int_a^b (r\omega)\left[\rho\left(l\cos\left(\left(\tan^{-1}\frac{r\omega}{V_w}\right)+\theta\right)\right)\left(\sqrt{(r\omega)^2+V_w^2}\right)\right]\ldots$$

$$\ldots\left(\left(\sqrt{(r\omega)^2+V_w^2}\right)\sin\left(2\theta+\left(\tan^{-1}\frac{r\omega}{V_w}\right)\right)-(r\omega)\right)dr$$

Finding Maximum Power

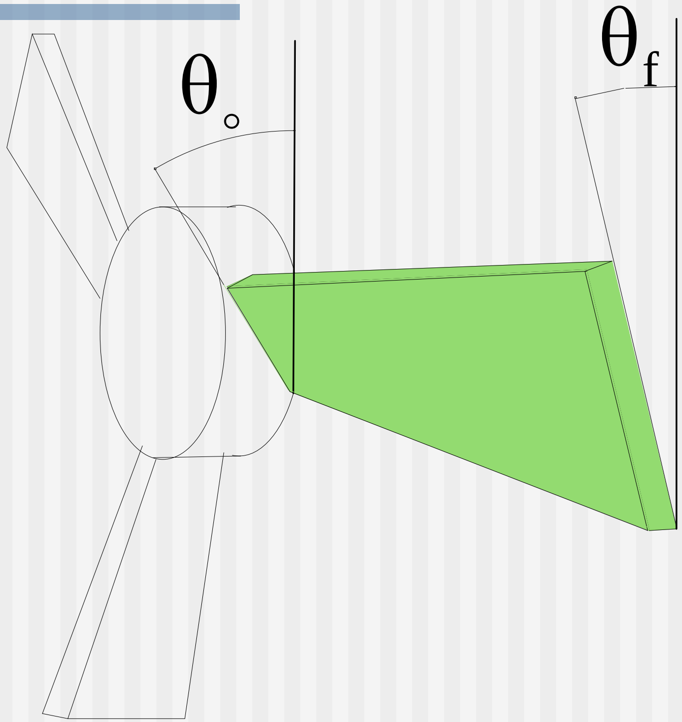
- MATLAB programming
- Several types of search methods:
  - visual, comparative, simplex
- Equation refinement (θ and ω)

θ dependence on r:

$$\theta = \theta_{\circ} e^{-3r/2L}$$

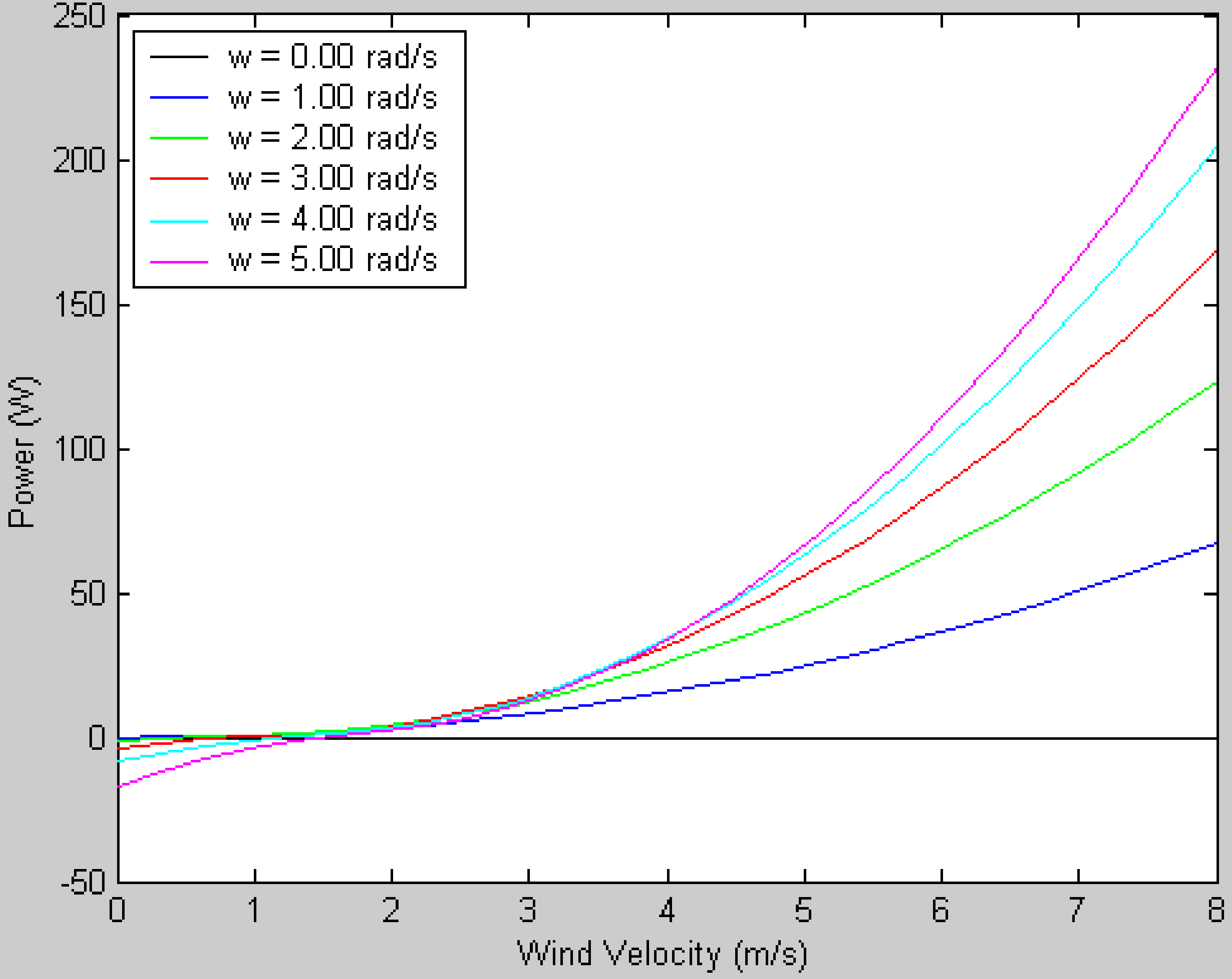
ω dependence on V:

$$\omega = \omega_{\circ} V$$

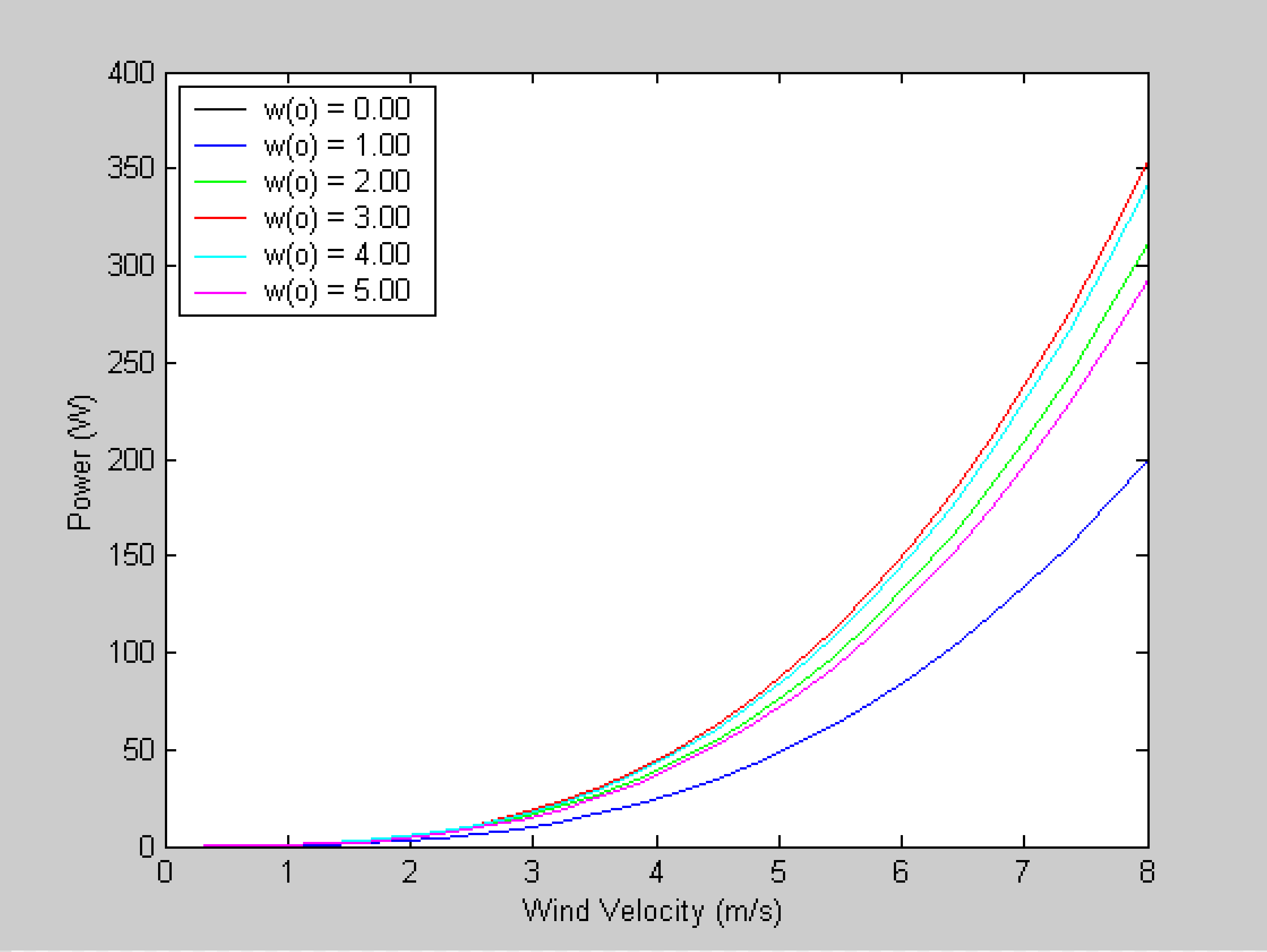


Constant values: ρ = 1.21kg/m^3,
L = 1.5m,
a = 0.1m, b = 1.5m

Angular velocity and theta are constant ($\theta = 18^{O}$).



Angular velocity varies with wind velocity ($\omega = \omega_o V$), and theta varies exponentially with r ($\theta = q \exp(-2r/3L)$, q = 18).



Improvements yet to be done:

- include frictional forces and airfoil shape
- vary blade width
- compare power curves to known turbine performances

Questions?